[Gr.G] The general graphics section (20 points)

Problem Gr.G-1. Color spaces (5 points)

Name two advantages of representing a color using CIE XYZ coordinates (over other three-coordinate representations).

Problem Gr.G-2. Filter convolution (5 points)

Sketch the shape that results from convolving a 2D box filter with itself.

Problem Gr.G-3. Transformations (5 points)

How many degrees of freedom are there in a projective mapping from 2D to 2D? From 3D to 3D? Defend your answers.

Problem Gr.G-4. CAGD (5 points)

Consider the three points $A = (1, 0)$, $B = (0, 0)$, and $C = (0, 1)$ in the plane. If $F([0\,..\,1])$ is a quadratic, polynomial Bézier curve with A, B, and C as its three Bézier points, what are the x and y coordinates of the point $F(1/2)$? What kind of conic section is F? On the other hand, suppose that the three sites A with weight 2, B with weight 1, and C with weight 2 are the Bézier sites of a quadratic, rational Bézier curve $G([0\,..\,1])$. What are the X and Y coordinates of $G(1/2)$? What kind of conic section is G?

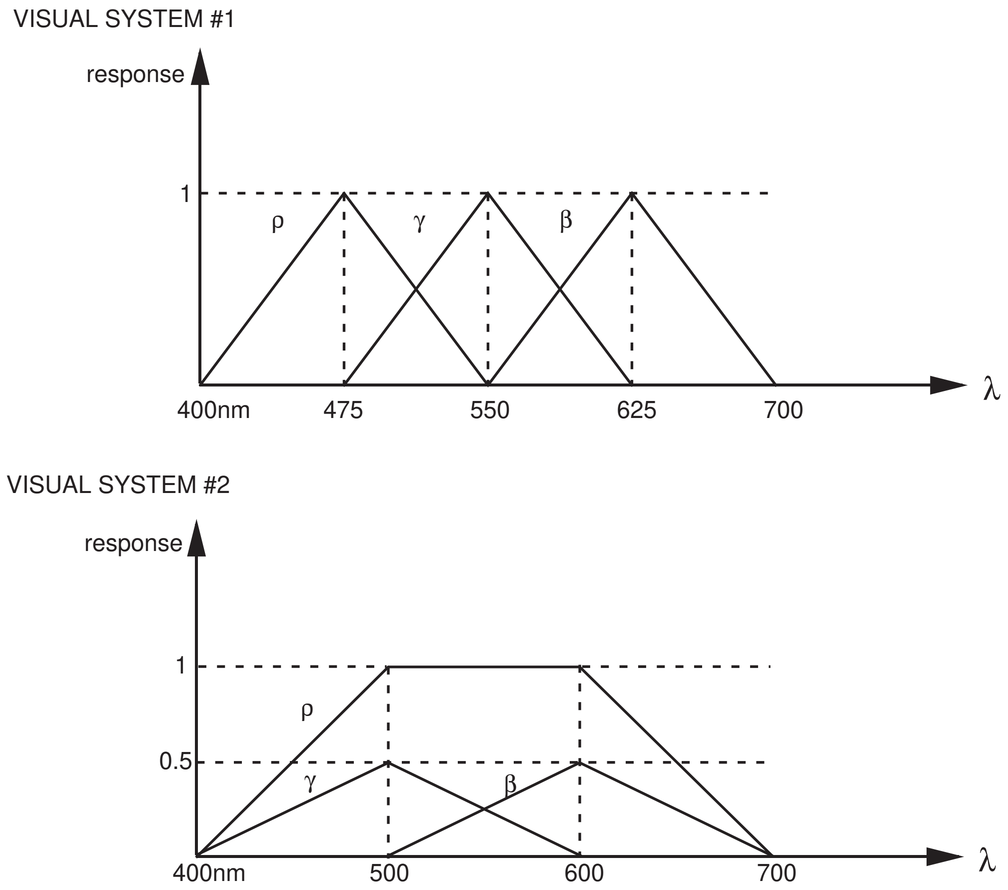

Figure 1: The two visual systems

[Gr.S] The specialized graphics section (40 points)

Problem Gr.S-1. Color spaces (10 points)

Consider the two hypothetical three-receptor visual systems described by the above sets of response curves.

(a) (5 points) Sketch in (ρ, γ, β)-space (a carefully labeled drawing will suffice) the locus of spectral colors corresponding to each visual system.

(b) (5 points) A visual system is called n-chromatic if the dimensionality of the space of colors that it can perceive is n. What is n for each of these visual systems? Defend your answer.

Problem Gr.S-2. Transformations (15 points)

A 2D rotation can be factored into four transformations: a Y-scale followed by a Y-shear followed by an X-scale followed by an X-shear. Give the sequence of four 2×2 transformation matrices, each of which implement one of the above transformations, that when concatenated together will rotate a 2D object by 45 degrees counterclockwise.

Problem Gr.S-3. Global illumination (15 points)

The "classical" radiosity formulation (i.e. the method of weighted residuals with point collocation and constant basis functions) leads to a linear system $\boldsymbol{e} = \boldsymbol{M}\boldsymbol{b}$ for patch emittances $\boldsymbol{e} = E_i$, patch radiosities $\boldsymbol{b} = B_i$, and radiosity matrix $\boldsymbol{M} = \boldsymbol{I} - \boldsymbol{K}$. In this equation, $\boldsymbol{I}$ is the identity matrix and $\boldsymbol{K} = \rho_i F_{i,j}$, where the ρ's and F's are reflectivities and form factors, respectively.

- **(a)** (8 points) Write down as many properties of the matrix $\boldsymbol{M}$ as you can think of. A terse list is fine.
- **(b)** (7 points) Consider a scene consisting of two empty adjoining rooms connected by a door. Given matrix $\boldsymbol{M}$ for this scene but no other information, how would you determine if the door is closed, preventing lighting from traveling between the two rooms? Assume that all patches have non-zero reflectivity, but assume no other knowledge about the scene and no knowledge of which matrix rows or columns correspond to which patches.