CS 242

Data Abstraction and Modularity

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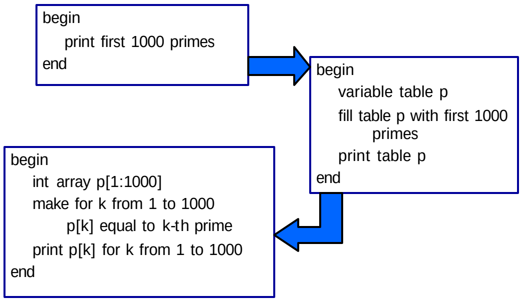
Topics

- Modular program development
  - Step-wise refinement
  - Interface, specification, and implementation
- Language support for modularity
  - Procedural abstraction
  - Abstract data types
    - Representation independence
    - Datatype induction
  - Packages and modules
  - Generic abstractions
    - Functions and modules with type parameters

Stepwise Refinement

- Wirth, 1971
  - "… program … gradually developed in a sequence of refinement steps"
  - In each step, instructions … are decomposed into more detailed instructions.
- Historical reading on web (CS242 Reading page)
  - N. Wirth, Program development by stepwise refinement, *Communications of the ACM,* 1971
  - D. Parnas, On the criteria to be used in decomposing systems into modules, *Comm ACM,* 1972
  - Both *ACM Classics of the Month*

Dijkstra's Example (1969)

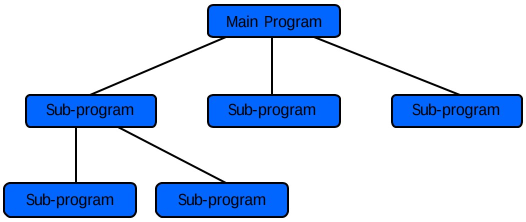


```
begin
    print first 1000 primes
end
```

```
begin
    variable table p
    fill table p with first 1000
        primes
    print table p
end
```

```
begin
    int array p[1:1000]
    make for k from 1 to 1000
        p[k] equal to k-th prime
    print p[k] for k from 1 to 1000
end
```

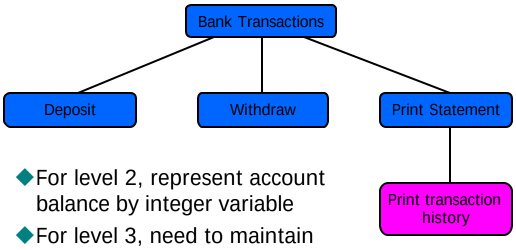
Program Structure



Data Refinement

- Wirth, 1971 again:
  - As tasks are refined, so the data may have to be refined, decomposed, or structured, and it is natural to refine program and data specifications in parallel

Example


Bank Transactions
Deposit
Withdraw
Print Statement
Print transaction history

- For level 2, represent account balance by integer variable
- For level 3, need to maintain list of past transactions

Modular program design

- Top-down design
  - Begin with main tasks, successively refine
- Bottom-up design
  - Implement basic concepts, then combine
- Prototyping
  - Build coarse approximation of entire system
  - Successively add functionality

Modularity: Basic Concepts

- Component
  - Meaningful program unit
    - Function, data structure, module, …
- Interface
  - Types and operations defined within a component that are visible outside the component
- Specification
  - Intended behavior of component, expressed as property observable through interface
- Implementation
  - Data structures and functions inside component

Example: Function Component

- Component
  - Function to compute square root
- Interface
  - float sqroot (float x)
- Specification
  - If x>1, then sqrt(x)* sqrt(x) ≈ x.
- Implementation

```
float sqroot  (float x){
   float y = x/2; float step=x/4; int i;
   for (i=0; i<20; i++){if ((y*y)<x) y=y+step; else y=y-step; step = step/2;}
   return y;
}
```

Example: Data Type

- Component
  - Priority queue: data structure that returns elements in order of decreasing priority
- Interface
  - Type pq
  - Operations empty : pq
    insert : elt * pq → pq
    deletemax : pq → elt * pq
- Specification
  - Insert add to set of stored elements
  - Deletemax returns max elt and pq of remaining elts

Heap sort using library data structure

- Priority queue: structure with three operations
  empty : pq
  insert : elt * pq → pq
  deletemax : pq → elt * pq
- Algorithm using priority queue (heap sort)

```
begin
  empty pq s
  insert each element from array into s
  remove elements in decreasing order and place in array
end
```

This gives us an O(n log n) sorting algorithm (see HW)

Language support for info hiding

- ◆Procedural abstraction
  - Hide functionality in procedure or function
- ◆Data abstraction
  - Hide decision about representation of data structure and implementation of operations
  - Example: priority queue can be binary search tree or partially-sorted array

In procedural languages, refine a procedure or data type by rewriting it. Incremental reuse later with objects.

Abstract Data Types

- ◆Prominent language development of 1970's
- ◆Main ideas:
  - Separate interface from implementation
    - Example:
      - Sets have empty, insert, union, is_member?, ...
      - Sets implemented as ... linked list ...
  - Use type checking to enforce separation
    - Client program only has access to operations in interface
    - Implementation encapsulated inside ADT construct

Origin of Abstract Data Types

- ◆Structured programming, data refinement
  - Write program assuming some desired operations
  - Later implement those operations
  - Example:
    - Write expression parser assuming a symbol table
    - Later implement symbol table data structure
- ◆Research on extensible languages
  - What are essential properties of built-in types?
  - Try to provide equivalent user-defined types
  - Example:
    - ML sufficient to define list type that is same as built-in lists

Comparison with built-in types

- ◆Example: int
  - Can declare variables of this type x: int
  - Specific set of built-in operations +, -, *, ...
  - No other operations can be applied to integer values
- ◆Similar properties desired for abstract types
  - Can declare variables x : abstract_type
  - Define a set of operations (give interface)
  - Language guarantees that only these operations can be applied to values of abstract_type

Clu Clusters

```
complex = cluster is
                  make_complex, real_part, imaginary_part, plus, times
    rep = struct [ re, im : real]
    make_complex = proc (x,y : real) returns (cvt)
         return (rep${re:x, im:y})
    real_part = proc (z:cvt) returns real
         return (z.re)
    imaginary_part = proc (z:cvt) returns real
         return (z.im)
    plus = proc (z, w: cvt) returns (cvt)
         return (rep${ re: z.re+w.re, im: z.im+w.im })
    mult = proc ...
end complex
```

ML Abstype

- ◆Declare new type with values and operations

```
abstype t = <tag> of <type>
    with
       val <pattern> =  <body>
          ...
       fun f(<pattern>) =  <body>
          ...
    end
```

- ◆Representation

`t = <tag> of <type>`   similar to ML datatype decl

Abstype for Complex Numbers

- Input

```
abstype cmplx = C of real * real with
   fun cmplx(x,y: real) = C(x,y)
   fun x_coord(C(x,y)) = x
   fun y_coord(C(x,y)) = y
   fun add(C(x1,y1), C(x2,y2)) = C(x1+x2, y1+y2)
end
```

- Types (compiler output)

```
type cmplx
val cmplx = fn : real * real -> cmplx
val x_coord = fn : cmplx -> real
val y_coord = fn : cmplx -> real
val add = fn : cmplx * cmplx -> cmplx
```

Abstype for finite sets

- Declaration

```
abstype 'a set = SET of 'a list with
   val empty = SET(nil)
   fun insert(x, SET(elts)) = ...
   fun union(SET(elts1), Set(elts2)) = ...
   fun isMember(x, SET(elts)) = ...
end
```

- Types (compiler output)

```
type 'a set
val empty = - : 'a set
val insert = fn : 'a * ('a set) -> ('a set)
val union = fn : ('a set) * ('a set) -> ('a set)
val isMember = fn : 'a * ('a set) -> bool
```

Encapsulation Principles

- Representation Independence
  - Elements of abstract type can be implemented in various ways
  - Restricted interface -> client program cannot distinguish one *good* implementation from another
- Datatype Induction
  - Method for reasoning about abstract data types
  - Relies on separation between interface and implementation

Representation Independence

- Integers
  - Can represent 0,1,2, ..., -1,-2, ... any way you want
  - As long as operations work properly
    +, -, *, /, print, ...
  - Example
    1's complement vs. 2's complement
- Finite Sets
  - can represent finite set {x, y, z, ... } any way you want
  - As long as operations work properly
    empty, ismember?, insert, union
  - Example
    linked list vs binary tree vs bit vector

Reality or Ideal?

- In Clu, ML, ... rep independence is a theorem
  - Can be proved because language restricts access to implementation: access through interface only
- In C, C++, this is an ideal
  - "Good programming style" will support representation independence
  - The language does not enforce it
    Example: print bit representation of -1
    This distinguishes 1's complement from 2's complement

Induction (Toward Datatype Induction)

- Main idea
  - 0 is a natural number
  - if x is a natural number, then x+1 is a natural number
  - these are all the natural numbers
- Prove p(n) for all n
  - prove p(0)
  - prove that if p(x) then p(x+1)
  - that's all you need to do

Skip: Will not cover datatype induction in any depth this year

Induction for integer lists

- Principle
  - nil is a list
  - if y is a list and x is an int, then cons(x,y) is a list
  - these are all of the lists
- Prove p(y) for all lists y
  - prove p(nil)
  - prove that if p(y) then p(cons(x,y))
  - that's all you need to do
- Example: next slide
  - Note: we do not need to consider car, cdr
  - Why? No new lists. (No subtraction in integer induction.)

Example of list induction

- Function to sort lists
  - fun sort(nil) = nil
  - | sort(x::xs) = insert(x, sort(xs))
- Insertion into sorted list
  - fun insert(x, nil) = [x]
  - | insert(x, y::ys) = if x<y then x::(y::ys)
  - else y::insert(x,ys)
- Prove correctness of these functions
  - Use induction on lists (easy because that's how ML let's us write them)

Interfaces for Datatype Induction

- Partition operations into groups
  - constructors: build elements of the data type
  - operators: combine elements, but no "new" ones
  - observers: produce values of other types
- Example:
  - sets with
    empty : set
    insert : elt * set -> set
    union : set * set -> set
    isMember : elt * set -> bool
  - partition
    construtors: empty, insert
    operator: union observer: isMember

Induction on constructors

- Operator: produces no new elements
  - Example: union for finite sets
    Every set defined using union can be defined without union:
    union(empty, s) = s
    union(insert(x,y), s) = insert(x, union(y,s))
- Prove property by induction
  - Show for all elements produced by constructors
    Set example: Prove P(empty) and P(y) => P(insert(x,y))
  - This covers all elements of the type

Example in course reader: equivalence of implementations

Example of set induction

- Assume map function
  - map(f,empty) = empty
  - map(f, insert(y,s)) = union(f(y), map(f,s))
- Function to find minimum element of list
  - fun intersect(s,s') = if empty(s') then s'
  - else let f(x) = if member(x,s) then {x} else empty
  - in map(f, s') end;
- Prove that this work:
  - Use induction on s':
    - Correct if s' = empty
    - Correct if s' = insert(y, s'')

What's the point of all this induction?

- Data abstraction hides details
- We can reason about programs that use abstract data types in an abstract way
  - Use basic properties of data type
  - Ignore way that data type is implemented
- This is not a course about induction
  - We may ask some simple questions
  - You will not have to derive any principle of induction

Modules

- General construct for information hiding
- Two parts
  - Interface:
    A set of names and their types
  - Implementation:
    Declaration for every entry in the interface
    Additional declarations that are hidden
- Examples:
  - Modula modules, Ada packages, ML structures, ...

Modules and Data Abstraction

```
module Set
   interface
      type set
      val empty : set
      fun insert : elt * set -> set
      fun union : set * set -> set
      fun isMember : elt * set -> bool
   implementation
      type set = elt list
      val empty = nil
      fun insert(x, elts) = ...
      fun union(...) = ...
      ...
end Set
```

- Can define ADT
  - Private type
  - Public operations
- More general
  - Several related types and operations
- Some languages
  - Separate interface and implementation
  - One interface can have multiple implementations

Generic Abstractions

- Parameterize modules by types, other modules
- Create general implementations
  - Can be instantiated in many ways
- Language examples:
  - Ada generic packages, C++ templates, ML functors, ...
  - ML geometry modules in course reader
  - C++ Standard Template Library (STL) provides extensive examples

C++ Templates

- Type parameterization mechanism
  - template<class T> ... indicates type parameter T
  - C++ has class templates and function templates
    - Look at function case now
- Instantiation at link time
  - Separate copy of template generated for each type
  - Why code duplication?
    - Size of local variables in activation record
    - Link to operations on parameter type

Example

- Monomorphic swap function

```
void swap(int& x, int& y){
      int tmp = x;  x = y;  y = tmp;
}
```

- Polymorphic function template

```
template<class T>
void swap(T& x, T& y){
     T tmp = x;  x = y;  y = tmp;
}
```

- Call like ordinary function

```
float a, b;  ...  ;  swap(a,b); ...
```

Generic sort function

- Function requires < on parameter type

```
template <class T>
void sort(  int count, T * A[count] ) {
   for (int i=0; i<count-1; i++)
       for (int j=I+1; j<count-1; j++)
            if (A[j] < A[i]) swap(A[i],A[j]);
}
```

- How is function < found?
  - Link sort function to calling program
  - Determine actual T at link time
  - If < is defined on T, then OK else error
    - May require overloading resolution, etc.

Compare to ML polymorphism

```
fun insert(less, x, nil) = [x]
|    insert(less, x, y::ys) = if less(x,y) then x::y::ys
                              else y::insert(less,x,ys)
fun sort(less, nil) = nil
|   sort(less, x::xs) = insert(less, x, sort(less,xs))
```

- Polymorphic sort function
  - Pass operation as function
  - No instantiation since all lists are represented in the same way (using cons cells like Lisp).
- Uniform data representation
  - Smaller code, can be less efficient, no complicated linking

Standard Template Library for C++

- Many generic abstractions
  - Polymorphic abstract types and operations
- Useful for many purposes
  - Excellent example of *generic programming*
- Efficient running time (but not always space)
- Written in C++
  - Uses template mechanism and overloading
  - Does *not* rely on objects

Architect: Alex Stepanov

Main entities in STL

- Container: Collection of typed objects
  - Examples: array, list, associative dictionary, ...
- Iterator: Generalization of pointer or address
- Algorithm
- Adapter: Convert from one form to another
  - Example: produce iterator from updatable container
- Function object: Form of closure ("by hand")
- Allocator: encapsulation of a memory pool
  - Example: GC memory, ref count memory, ...

Example of STL approach

- Function to merge two sorted lists
  - merge : range(s) × range(t) × comparison(u)

    → range(u)

  This is conceptually right, but not STL syntax.
- Basic concepts used
  - range(s) - ordered "list" of elements of type s, given by pointers to first and last elements
  - comparison(u) - boolean-valued function on type u
  - subtyping - s and t must be subtypes of u

How merge appears in STL

- Ranges represented by iterators
  - iterator is generalization of pointer
  - supports ++ (move to next element)
- Comparison operator is object of class Compare
- Polymorphism expressed using template

```
template < class InputIterator1, class InputIterator2,
           class OutputIterator, class Compare >
OutputIterator merge(InputIterator1 first1, InputIterator1 last1,
             InputIterator2 first2, InputIterator1 last2,
               OutputIterator result, Compare comp)
```

Comparing STL with other libraries

- C:

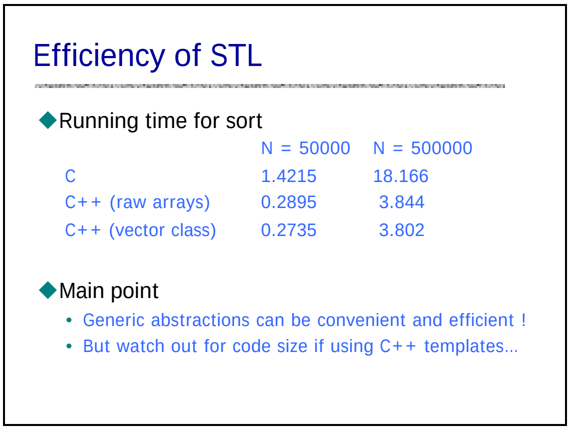
  `qsort( (void*)v, N, sizeof(v[0]), compare_int );`
- C++, using raw C arrays:

  `int v[N];`

  `sort( v, v+N );`
- C++, using a vector class:

  `vector v(N);`

  `sort( v.begin(), v.end() );`



Efficiency of STL

- Running time for sort

	N = 50000	N = 500000
C	1.4215	18.166
C++ (raw arrays)	0.2895	3.844
C++ (vector class)	0.2735	3.802

- Main point
  - Generic abstractions can be convenient and efficient !
  - But watch out for code size if using C++ templates…