

**Computer Science Department
Stanford University
Comprehensive Examination in Numerical Analysis
Autumn 1991**

October 9, 1991

READ THIS FIRST!

1. You should write your answers for this part of the Comprehensive Examination in a **BLUE BOOK**. Be sure to write your **MAGIC NUMBER** on the cover of every blue book that you use.
2. The number of **POINTS** for each problem indicates how elaborate an answer is expected. For example, an essay-type question worth 6 points or less doesn't deserve an extremely detailed answer, even though a person can expound at length on just about any topic in computer science.
3. The total number of points is 30, and the exam takes 30 minutes. This "coincidence" can help you plan your time.
4. This exam is **OPEN BOOK**. You may use notes, articles, or books—but no help from other sentient agents such as other humans or robots.
5. Show your work, since **PARTIAL CREDIT** will be given for incomplete answers. For example, you can get credit for making a reasonable start on a problem even if the idea doesn't work out; you can also get credit for realizing that certain approaches are incorrect. On a true/false question, you might get partial credit for explaining why you think something is true when it is actually false. But no partial credit can be given if you write nothing.

Problem I:(16 points). **Linear Systems and Matrices.**

Consider the matrix

$$A = \begin{bmatrix} 4 & -2 & 1 \\ -2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}.$$

- (a) (3 points). Show that the LU factorization of A does not exist. Give necessary and sufficient conditions for the LU factorization to exist.
- (b) (4 points). Show how a permutation can be introduced to obtain a matrix B for which the LU factorization exists. Compute the LU factorization of this matrix B .
- (c) (4 points). Show how the LU factorization can be used to compute the third column of A^{-1} without computing A^{-1} completely.
- (d) (5 points). Suppose that the function $f(x)$ was measured in 4 points with the following results:

x	-2	-1	0	1
$f(x)$	-1	1	1	4

We want to approximate $f(x)$ by $p_2(x) = a + bx + cx^2$ with the method of least squares. Establish the corresponding linear system of equations and show that it does not possess a solution.

Problem II:(14 points). **Interpolation.**

- (a) (5 points). Construct the polynomial

$$p_2(x) = a_2x^2 + a_1x + a_0$$

of degree 2 which interpolates the function

$$f(x) = \frac{1}{1+x}$$

at the points $x_0 = 0, x_1 = \frac{1}{2}, x_2 = 1$.

- (b) (5 points). Give an upper bound for the interpolation error $|f(x) - p_2(x)|$ on $[0, 1]$.
- (c) (4 points). Show that the polynomial $p_n(x)$ of degree $\leq n$ interpolating a function $f(x)$ in $n + 1$ points $x_0 < x_1 < \dots < x_n$ is unique.