

2007 Comprehensive Examination Solutions

Logic

1. Logic. (20 points)

- (a) True
- (b) False
- (c) True
- (d) False
- (e) False
- (f) True
- (g) False
- (h) False
- (i) False
- (j) True

2. Unification. (10 points)

- (a) $\{w \leftarrow r(z, z), x \leftarrow t(r(z, z), z), y \leftarrow v\}$
- (b) $p(t(t(r(z, z), z), v), r(z, z))$
- (c) $\{x \leftarrow y\}$ and $\{y \leftarrow x\}$ are most general unifiers of $p(x)$ and $p(y)$.

3. Clausal Form. (10 points)

- $\{\neg p(z, f(z)), p(y, b)\}$
- $\{\neg p(z, a), p(y, b)\}$

4. Resolution. (10 points)

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| 1. $\{\neg p(x, y), q(x, y, f(x, y))\}$ | Premise |
| 2. $\{\neg r(y, z), q(a, y, z)\}$ | Premise |
| 3. $\{r(y, z), \neg q(a, y, z)\}$ | Premise |
| 4. $\{p(x, g(x)), q(x, g(x), z)\}$ | Premise |
| 5. $\{\neg r(x, y), \neg q(x, w, z)\}$ | Premise |
| 6. $\{\neg q(a, x, y), \neg q(x, w, z)\}$ | 3, 5 |
| 7. $\{q(x, g(x), f(x, g(x))), q(x, g(x), z)\}$ | 1, 4 |
| 8. $\{\neg q(g(a), w, z)\}$ | 6, 7 (factoring 7) |
| 9. $\{\}$ | 7, 8 (factoring 7) |

5. Model Building. (10 points)

- (a) $p^i = \{\}$
(b) $p^i = \{\text{john}\}$ and $q^i = \{\text{john}\}$

6. Herbrand Models. (10 points) The following sentence is satisfiable but has no Herbrand model.

$$r(a) \wedge \exists x. \neg r(x)$$

7. Theory Completeness. (20 points)

- (a) Incomplete
(b) Incomplete
(c) Complete
(d) Complete
(e) Incomplete
(f) Complete
(g) Complete
(h) It is not complete but it is maximal in that every sentence in the universal language is either logically entailed or inconsistent.