COMPUTER SCIENCE DEPARTMENT
STANFORD UNIVERSITY
COMPREHENSIVE EXAMINATION IN NUMERICAL ANALYSIS
FALL 2006

1. Newton's Method. Suppose we want to compute the reciprocal of a number on a computer which does not have division.

(a) Consider the equation

$$f(x) = \frac{1}{x} - a, \qquad a \geq 0.$$

Show how Newton's method can be used for computing $1/a$. What is your initial choice of x_0?

(b) Discuss the convergence properties of the algorithm. Show the behavior of

$$x_k - \frac{1}{a}.$$

(c) Suppose A is an $n \times n$ matrix of rank n.

(i) Show how Newton's method can be used for computing A^{-1}. What is the initial choice of X_0?

(ii) How many operations are needed at each iteration?

2. Interpolation.

(a) Let $p_n(x)$ and $q_n(x)$ be polynomials of degree n. Assume that

$$p_n(x_i) = y_i, \qquad i = 0, 1, \ldots, n,$$
$$q_n(x_i) = y_i, \qquad i = 0, 1, \ldots, n,$$

and $x_1, \ldots, x_n$ are distinct. Show that $p_n(x) \equiv q_n(x)$.

(b) Give a definition of a cubic spline. Explain the benefits of using a spline rather than using classical interpolation. What are the computational costs associated with a cubic spline?

3. Differential equations. Consider the differential equation

$$y' = \lambda y$$

with

$$y(0) = 1.$$

Assume that $\lambda < 0$. Construct the following numerical methods for solving this equation:

Euler: $y_{k+1} = y_k + hy'_k$.
Backward Euler: $y_{k+1} = y_k + hy'_{k+1}$.

(a) Discuss the stability of each method as $k \to \infty$.

(b) Give the advantages and disadvantages of using a Runge-Kutta method as opposed to a multistep method.