Computer Graphics Comprehensive Exam

Computer Science Department
Stanford University
Fall 2006

NAME:

Note: This is exam is *closed-book.*

The exam consists of 5 questions. Each question is worth 20 points. Please answer all the questions in the space provided, overflowing on to the back of the page if necessary.

You have 60 minutes to complete the exam.

1. [20 points] Rotations

A 2D rotation about the origin by θ is represented by the following matrix:

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

1A [10 points] Show that $R(\theta) \bullet R(\varphi) = R(\theta + \varphi)$. That is, show that multiplying two rotation matrices yields another rotation matrix; the angle of rotation of the resulting matrix is the sum of the angle of rotations of the two product matrices.

1B [10 points] Show that $R^{-1}(\theta) = R(-\theta)$. That is, show that the inverse of the rotation matrix is equal to a rotation in the opposite direction. In order to solve this problem, you need to find the inverse of the rotation matrix.

2. [20 points] Color coordinates.

2A [5 points]. Why do displays such as cathode-ray tubes and liquid crystal displays use three color primaries, red, green and blue? That is, why don't they need more colors?

2B [5 points]. Why do color-laser and ink-jet printers use 4 or more colors? For example, many printers use cyan, magenta, yellow and black inks, and high-end printers often use 6 or more colors.

2C [10 points]. What is an (1) *absolute* color coordinate system, a (2) *relative* color coordinate system, and an (3) *intuitive* or artistic color coordinate system. Give a one sentence definition of each type of coordinate system.

3. [20 points] Texture mapping.

Texture mapping is one of the most important operations in a modern 3D graphics system. Textures are images that are applied to a surface. The basic texture map drawing loop is:

```
for (x,y) inside triangle
    (u,v) = TransformFromPixelSpaceToTextureSpace(x,y)
    color = TextureLookup(texture, u, v)
```

Here, (x,y) is the location of a pixel inside the triangle being drawn, and (u,v) is the location of the pixel in texture coordinates. Assume pixel and texture coordinates are floating point numbers, and that integral values fall on pixel and texel centers. For example, (0.0,0.0) is the first pixel/texel, (1.0,0.0) is the second pixel/texel, etc.

3A [5 points] Images and textures are represented as images, and images are represented as 2D arrays of values. Yet, texture coordinates have higher precision. That is, texture coordinates are not necessarily integers. Why should texture coordinates have additional precision?

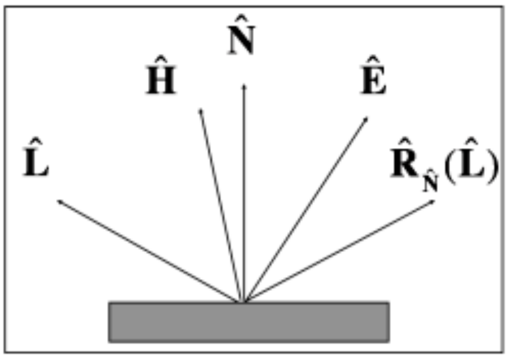
3B [5 points] One way to implement `TextureLookup` is by using *nearest-neighbor interpolation*. Write pseudo-code to implement nearest neighbor interpolation.

3C [5 points] A better way to implement `TextureLookup` is by using *bilinear interpolation*. Write pseudo-code to implement bilinear interpolation.

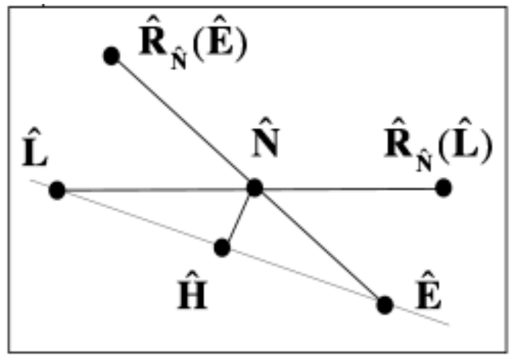
3D [5 points] Bilinear interpolation is much more expensive than nearest-neighbor interpolation, yet all high quality implementation of texture mapping include this type of interpolation. Why, according to theory, is it preferable to use bilinear interpolation?

4. [20 points] Highlights.

The following figure shows the geometry of reflection. $\hat{\mathbf{N}}$ is a unit vector normal to the surface at the point of reflection. $\hat{\mathbf{L}}$ is a unit vector from the surface to the light. $\hat{\mathbf{E}}$ is a unit vector from the surface to the eye. And, $\hat{\mathbf{H}}$ is the unit halfway vector $\hat{\mathbf{H}} = \frac{\hat{\mathbf{L}} + \hat{\mathbf{E}}}{\left|\hat{\mathbf{L}} + \hat{\mathbf{E}}\right|}$.



side view



top view

There are two other vectors of interest. These are the reflections of $\hat{\mathbf{L}}$ and $\hat{\mathbf{E}}$ about the normal. These vectors may be computed with the reflection transformation, $\hat{\mathbf{R}}_{\hat{\mathbf{N}}}(\mathbf{V}) = \mathbf{V} - 2\left(\hat{\mathbf{N}} \bullet \mathbf{V}\right)\mathbf{V}$, which reflects the vector $\mathbf{V}$ about the normal $\hat{\mathbf{N}}$. With these definitions, the Phong model is $\left(\hat{\mathbf{R}}_{\hat{\mathbf{N}}}(\hat{\mathbf{L}}) \bullet \hat{\mathbf{E}}\right)^s$ and the Blinn model is $\left(\hat{\mathbf{H}} \bullet \hat{\mathbf{N}}\right)^s$.

4A [5 points]. The parameter *s* is controlled by the application. What is the significance of this parameter. What happens when you change s?

4B [15 points]. Suppose $\hat{\mathbf{N}}$, $\hat{\mathbf{L}}$, and $\hat{\mathbf{E}}$ are in the same plane. What is the relationship between $\left(\hat{\mathbf{R}}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}}) \bullet \hat{\mathbf{L}}\right)^s$ and $\left(\hat{\mathbf{H}} \bullet \hat{\mathbf{N}}\right)^s$. Hint, is there a relationship between the angles between the various vectors?

5 [20 points] Visualization.

Your friend Sally the brilliant mathematics graduate student asks for your help because of your expertise in computer graphics. She wants to visualize the function

$$f(x,y,z) = 1 - x^2 + y^2 + z^2 + \sin 4x + \sin 4y + \sin 4z$$

5A [15 points]. How would you make a picture of her function? Assume you have access to a basic graphics library, but not an advanced visualization system such as Matlab, Mathematica or a graphing calculator. That is, break down the visualization process into standard algorithms and basic graphic primitives. Outline the method you would use. Make sure to include enough detail about each method to ensure that they work properly.

5B [5 points]. Does your picture show the important properties of her function? What features might be of interest, and how would you guarantee that those features were clear in your picture?