Automata and Formal Languages Comprehensive Exam Solutions

Fall 2006

Problem 1 (20 points)

(a) True. See Theorem 4.4 of HMU (page 132) for a proof.

(b) True. See Theorem 4.8 of HMU (page 135) for a proof.

(c) True. Let A and B be CFLs, say generated by grammars G_1 and G_2 with start symbols S_1 and S_2, respectively. Form a grammar generating $A \cup B$ by combining all productions of G_1 and G_2 along with a new start symbol S and new productions $S \to S_1$ and $S \to S_2$. (Also, before combining the grammars, make sure that they have disjoint sets of nonterminals, renaming the nonterminals of one grammar if needed.)

(d) False. For a counterexample, see Examples 7.19 and 7.26 of HMU.

Problem 2 (12 points)

Classify each of the following languages as being in one of the following classes of languages: *empty, finite, regular, context-free, recursive, recursively enumerable, all languages.*

(a) Recursive.

(b) All languages (as the set of all strings is recursive).

(c) Context-free.

(d) Recursive.

Problem 3 (18 points)

(a) See Theorem 9.8 in HMU (page 385) for a proof.

(b) See Theorem 9.9 in HMU (page 386) for a proof.

Problem 4 (20 points)

We give a polynomial-time reduction from CLIQUE to SUBGRAPH ISOMORPHISM. Since CLIQUE is NP-complete, this proves that SUBGRAPH ISOMORPHISM is NP-hard.

We begin with an instance of CLIQUE, specified by a graph G and a positive integer k. We construct an instance of SUBGRAPH ISOMORPHISM by defining G_1 to be the graph G and the graph G_2 to be a complete graph on k nodes. This reduction clearly runs in polynomial time.

Suppose G has a clique of size k. Then this same subset of nodes (viewed as a subgraph of G_1) is isomorphic to G_2. Conversely, if G_2 is isomorphic to a subgraph of G_1, then since G_2 is a complete graph on k nodes, this subset of nodes of G_1 corresponds to a clique of size k in G. These two facts establish the correctness of the reduction.