Automata and Formal Languages Comprehensive Exam (60 Points)

Fall 2005

Problem 1 (10 points)

(a) Show that if L is a regular language, then so is

$$min(L) = \{w : w \text{ is in } L\text{, but no proper prefix of } w \text{ is in } L\}.$$

(b) Show that if L is a regular language, then so is

$$init(L) = \{w : \text{there exists an } x \text{ such that } wx \text{ is in } L\}.$$

Hint: For each part, start with a DFA for L and modify it. Include *brief* arguments justifying your constructions.

Problem 2 (15 points)

Decide whether the following statements are TRUE or FALSE. *You will receive 3 points for each correct answer and -2 points for each incorrect answer.*

(a) There is a language L such that neither L nor its complement are recursively enumerable.

(b) Suppose there is a polynomial-time reduction from the language L_1 to the language L_2. If L_1 is NP-hard, then L_2 must be NP-complete.

(c) Suppose there is a polynomial-time reduction from the language L_1 to the language L_2. If L_1 is NP-complete, then L_2 must be NP-hard.

(d) The following language is recursively enumerable: encodings of Turing machines that accept at least 154 different inputs.

(e) The following language is recursively enumerable: encodings of Turing machines that accept at most 154 different inputs.

Problem 3 (15 points)

Classify each of the following languages as being in one of the following classes of languages: *empty, finite, regular, context-free, recursive, recursively enumerable, all languages.* You must give the *smallest* class that contains *every possible language* fitting the following definitions. For example, the language of a DFA could be empty or finite, and must always be context-free, but the smallest class that contains all such languages is that of the *regular* languages. *You will receive 3 points for each correct answer and -2 points for each incorrect answer.*

(a) A subset of a regular language.

(b) The concatentation of two recursively enumerable languages. (Recall that the concatenation of languages L_1 and L_2 is $L_1L_2 = \{wx \mid w \in L_1, x \in L_2\}$.)

(c) The concatentation of two recursively enumerable languages, one of which is the complement of the other.

(d) An NP-complete language.

(e) An NP-hard language.

Problem 4 (20 points)

An instance of the INTEGER LINEAR PROGRAMMING PROBLEM is the following: given a set of linear constraints of the form $\sum_{i=1}^{n} a_i x_i \leq c$ or $\sum_{i=1}^{n} a_i x_i \geq c$, where the a's and c's are integer constants and x_1, x_2, $\ldots$, x_n are variables, does there exist an assignment of integers to each of the variables that makes all of the the constraints true? Prove that the INTEGER LINEAR PROGRAMMING PROBLEM is NP-hard.