2005 Comprehensive Examination Solutions
Artificial Intelligence

1. Search. (20 points) Costs for finding a solution at depth k in a tree with branching factor 2 with overall depth d.

Time	*Best*	*Worst*
Depth – First Search	k	$2^d - 2^{d-k+1} + 1$
Breadth – First Search	2^{k-1}	$2^k - 1$
Iterative Deepening	$2^k - 1$	$2^{k+1} - k - 2$

Iterative deepening is simply a sum of breadth-first search at levels up to k. A key benefit is that it uses only the same amount of space as depth-first search.

2. Logic. (20 points)
(a) True
(b) False
(c) True
(d) False
(e) True
(f) False
(g) False
(h) False
(i) False
(j) True

3. Resolution. (20 points)

Clauses:
1. $\{\neg p(x,y,z), s(x,y,z)\}$
2. $\{\neg q(x,y,z), s(x,y,z)\}$
3. $\{p(x,f(x),z), q(x,f(x),z), \neg r(x)\}$
4. $\{r(a)\}$
5. $\{\neg s(a,y,b), \neg s(a,y,c)\}$

Derivation:
6. $\{pa,f(a),z), q(a,f(a),z)\}$ 3,4
7. $\{s(a,f(a),z), q(a,f(a),z)\}$ 1,6
8. $\{s(a,f(a),z)\}$ 2,7
9. $\{\neg s(a,f(a),c)\}$ 5,8
10. $\{\}$ 8,9

4. Bayes Nets. (20 points)

(a) We compute $p(G)$ by summing over all cases in which the desired condition is true.

$$p(G)=\sum_{a}\sum_{b}\sum_{c}\sum_{d}\sum_{e}\sum_{f} p(a,b,c,d,e,f,G)$$

(b) Using the chain rule for probability, we can write $p(a,b,c,d,e,f,G)$ as follows.

$$p(a,b,c,d,e,f,G)=p(a)\,p(b|a)\,p(c|a,b)\,p(d|a,b,c)\,p(e|a,b,c,d)\,p(f|a,b,c,d,e)\,p(G|a,b,c,d,e,f)$$

Using conditional independences from the Bayes net, we can rewrite this more compactly.

$$p(a,b,c,d,e,f,G)=p(a)\,p(b\mid a)\,p(c\mid a,e)\,p(d\mid b,c)\,p(e)\,p(f\mid e)\,p(G\mid d,f)$$

Inserting this version into the expression from part (a), we get the following.

$$p(G)=\sum_{a}\sum_{b}\sum_{c}\sum_{d}\sum_{e}\sum_{f} p(a)p(b\mid a)p(c\mid a,e)p(d\mid b,c)p(e)p(f\mid e)p(G\mid d,f)$$

Finally, we can move some of the factors outside of sums, saving some multiplications.

$$p(G)=\sum_{a} p(a)\sum_{b} p(b\mid a)\sum_{c}\sum_{d} p(d\mid b,c)\sum_{e} p(c\mid a,e)p(e)\sum_{f} p(f\mid e)p(G\mid d,f)$$

(c)

Variable	Factors Used	Factor Generated
B	$p(b \mid a)\ p(d \mid b, c)$	$g_1(a, c, d)$
C	$p(c \mid a, e)\ g_1(a, c, d)$	$g_2(a, d, e)$
D	$p(G \mid d, f)\ g_2(a,d, e)$	$g_3(a, e, f,G)$
F	$p(f \mid e)\ g_3(a, e, f, G)$	$g(a, e,G)$

5. Learning. (20 points)

(a) The concept is *a xor d*; so a 2 level tree is all that is needed.

(b) 1 bit

(c) 1 bit in either case.

(d) 0 bits for a and 1 bit for d given a.