Comprehensive Exam: Algorithms and Concrete Mathematics Autumn 2005

This is a one hour closed-book exam and the point total for all questions is 60.

In questions that ask you to provide an algorithm, please explain the algorithm in words and diagrams, no need to write code or pseudo code. Also, for any algorithm, state and prove its running time. No credit will be given for exponential-time algorithms. Polynomial but slow algorithms will get some partial credit. Amount of credit will depend on how much slower they are compared to what is achievable using the knowledge in the reading list.

For full credit, the answers should be short and precise. Long and convoluted answers will not get full credit even if they are correct.

The following is a statement of the Stanford University Honor Code:

A. *The Honor Code is an undertaking of the students, individually and collectively:*
   (1) *that they will not give or receive aid in examinations; that they will not give or receive unpermitted aid in class work, in the preparation of reports, or in any other work that is to be used by the instructor as the basis of grading;*
   (2) *that they will do their share and take an active part in seeing to it that others as well as themselves uphold the spirit and letter of the Honor Code.*

B. *The faculty on its part manifests its confidence in the honor of its students by refraining from proctoring examinations and from taking unusual and unreasonable precautions to prevent the forms of dishonesty mentioned above. The faculty will also avoid, as far as practicable, academic procedures that create temptations to violate the Honor Code.*

C. *While the faculty alone has the right and obligation to set academic requirements, the students and faculty will work together to establish optimal conditions for honorable academic work.*

By writing my "magic number" below, I certify that I acknowledge and accept the Honor Code.

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(Number)

Prob	# 1	# 2	# 3	# 4	Total
Score					
Max	10	20	15	15	60

1. **[10 pts]** Prove a tight asymptotic bound on the behavior of $T(n) = T(n-1) + \ln n$, where $T(1) = 1$.

2. **[20 pts]** Let $d > 0$ be a small integer. We study a heap structure where the heap branching factor is d rather than 2. We call such a heap a d-heap (the standard heap is a 2-heap). Note that a d-heap of depth ℓ has d^ℓ elements in it.

   a. Give an efficient algorithm for insert and extract-min for a d-heap. Give the asymptotic running time for your algorithm.

   b. Give the best algorithm you can think of for constructing a d-heap from a given vector of n elements (build-heap). Give the asymptotic running time for your algorithm.

   c. Describe a sorting algorithm that uses a d-heap. Give the asymptotic running time for your algorithm. The running time will depend on n (the size of the given list) and on d.

   d. What is the optimal value of d for getting the fastest sorting algorithm?

3. **[15 pts]** Let $G = (V, E)$ be a connected undirected graph. A bridge is an edge $e \in E$ such that removing e disconnects the graph, i.e. breaks the graph into at least two connected components. Give an $O(|E|)$ time algorithm to find all bridge edges of G.
   Hint: use DFS.

4. **[15 pts]** You are given an array of n objects. You are told that some element occurs at least $\lfloor n/2 \rfloor + 1$ times in the array. We call this element the majority element.

   Suppose the objects are totally ordered (that is you are given a function that takes two objects A and B as input and returns (key$(A) <$ key(B))). Give an algorithm that finds the majority element in time $\Theta(n)$.
   Hint: a one line answer is sufficient.