Automata and Formal Languages Comprehensive Exam

Fall 2004

Problem 1 (10 points)

Give context-free grammars generating the following languages over the alphabet $\{0, 1\}$ (you need not prove the correctness of your grammars):

(a) $\{a^i b^j a^{i+j+k} b^k : i, j, k \geq 0\}$;

(b) all strings with an equal number of a's and b's.

Problem 2 (15 points)

Decide whether the following statements are TRUE or FALSE. *You will receive 3 points for each correct answer and -2 points for each incorrect answer.*

(a) If L_1 and L_2 are both non-regular, then $L_1 \cap L_2$ must be non-regular.

(b) Suppose there is a polynomial-time reduction from the language L_1 to the language L_2. It is possible that L_1 is solvable in polynomial time but L_2 is not even in NP.

(c) Suppose there is a polynomial-time reduction from the language L_1 to the language L_2. If L_1 is recursive, then L_2 must be recursive.

(d) Every infinite regular set contains a subset that is not recursively enumerable.

(e) Every infinite recursively enumerable set contains an infinite subset that is recursive.

Problem 3 (15 points)

Classify each of the following languages as being in one of the following classes of languages: *empty, finite, regular, context-free, recursive, recursively enumerable.* You must give the *smallest* class that contains *every possible language* fitting the following definitions. For example, the language of a DFA could be empty or finite, and must always be context-free, but the smallest class that contains all such languages is that of the *regular* languages. *You will receive 3 points for each correct answer and -2 points for each incorrect answer.*

(a) The intersection of a context-free language and a regular language.

(b) The intersection of a recursive language and a regular language.

(c) The languages accepted by nondeterministic pushdown automata with a single state that accept by empty stack.

(d) The languages accepted by nondeterministic pushdown automata with two stacks.

(e) The complement of a language in NP.

Problem 4 (15 points)

Specify which of the following problems are *decidable* and which are *undecidable*. *You will receive 3 points for each correct answer and -2 points for each incorrect answer.*

(a) Given a Turing machine M, does M halt when started with an empty tape?

(b) Given a context-free language L and a regular language R, is $L \subseteq R$?

(c) Given a context-free language L and a regular language R, is $R \subseteq L$?

(d) Given a DFA, does it accept on only finitely many inputs?

(e) Given a PDA, does it accept on only finitely many inputs?

Problem 5 (15 points)

A *monotone* 2-SAT formula is a 2-CNF Boolean formula $F(x_1, \ldots, x_n)$ that does not contain negated variables. For example:

$$F(x_1, x_2, x_3, x_4) = (x_1 \vee x_2) \wedge (x_2 \vee x_4) \wedge (x_1 \vee x_4) \wedge (x_2 \vee x_3).$$

It is clear that there always exists a truth assignment for the variables $x_1, \ldots, x_n$ satisfying the formula F—simply set each variable to TRUE.

Consider the following problem called MONOTONE 2-SAT: given a monotone 2-SAT formula F and a positive integer k, determine whether there exists a truth assignment satisfying F such that the number of variables set to TRUE is *at most* k.

Prove that the MONOTONE 2-SAT problem is NP-complete. (**Hint:** Think about the NP-complete VERTEX COVER problem.)