Stanford University
Computer Science Department

Fall 2003 Comprehensive Exam in Artificial Intelligence

1. **Closed Book, No Laptop & Notes Write only in the Blue Book provided.**
2. **The exam is timed for one hour.**
3. **Write your Magic Number on this sheet & on the Blue Book.**

The following is a statement of the Stanford University Honor Code:

A. The Honor Code is an undertaking of the students, individually and collectively:

1. that they will not give or receive aid in examinations; that they will not give or receive unpermitted aid in class work, in the preparation of reports, or in any other work that is to be used by the instructor as the basis of grading;

2. that they will do their share and take an active part in seeing to it that others as well as themselves uphold the spirit and letter of the Honor Code.

B. The faculty on its part manifests its confidence in the honor of its students by refraining from proctoring examinations and from taking unusual and unreasonable precautions to prevent the forms of dishonesty mentioned above. The faculty will also avoid, as far as practicable, academic procedures that create temptations to violate the Honor Code.

C. While the faculty alone has the right and obligation to set academic requirements, the students and faculty will work together to establish optimal conditions for honorable academic work.

By writing my Magic Number below, I certify that I acknowledge and accept the Honor Code.

Magic Number---------------------------------

2003 Comprehensive Examination
Artificial Intelligence

1. Search. (20 points) Consider a search tree with uniform branching factor 2 and depth d, and consider a search problem for which there is a single solution in the tree at depth k. Give expressions for the worst case cost of finding the solution, in terms of nodes visited, for (a) breadth-first search, (b) depth-first search, and (c) iterative deepening starting at depth 0 and incrementing by 1 on each iteration. Comments on dealing with fence-posts: The root of the tree is at depth 0, and the cost of finding a solution includes the visit to the solution node.

2. Logic. (20 points) Let Γ and Δ be sets of sentences in first-order logic, and let φ and ψ be individual sentences in first-order logic. State whether each of the following statements is true or false. (No explanation is necessary.)

(a) If $\Gamma \vdash \varphi$ and $\Delta \vdash \varphi$, then $\Gamma \cup \Delta \models \varphi$.
(b) If $\Gamma \vdash \varphi$ and $\Delta \vdash \varphi$, then $\Gamma \cap \Delta \models \varphi$.
(c) If $\Gamma \vdash \varphi$ and $\Delta \vdash \psi$, then $\Gamma \cup \Delta \models (\varphi \Rightarrow \psi)$.
(d) If $\Gamma \vdash \varphi$ and $\Delta \vdash \psi$, then $\Gamma \cup \Delta \models (\neg \varphi \Rightarrow \psi)$.
(e) If $\Gamma \vdash \varphi$ and $\Delta \vdash \psi$, then $\Gamma \cap \Delta \models (\varphi \Rightarrow \psi) \vee (\psi \Rightarrow \varphi)$.

3. Automated Reasoning. (20 points) Use the resolution refutation method to prove $\forall x.(p(x) \Rightarrow r(x))$ from the following premises.

$$\forall x.(p(x) \Rightarrow \exists y.(q(x,y) \vee q(y,x)))$$
$$\forall x.\forall y.((q(x,y) \vee q(y,x)) \Rightarrow r(x))$$

Note that this is a question about the resolution refutation method. You will get zero points, nothing, nada, zip, no score for proving it in any other way.

4. Probability. (20 points) Suppose there are three chests, each with two drawers. The first chest has a gold coin in each drawer. The second has a gold coin in one drawer and a silver coin in the other drawer. The third chest has a silver coin in each drawer.

(a) Three of the six drawers are selected at random. What is the probability that all three drawers contain gold coins?
(b) A chest is chosen at random, and a drawer is opened. If the drawer contains a gold coin, what is the probability that the other drawer also contains a gold coin? Be careful.

5. Learning. (20 points) Consider the following training set for a decision-tree learning problem.

Example	*a*	*b*	*c*	*d*	*e*	**Goal**
x_1	1	0	0	0	0	1
x_2	1	1	1	1	0	0
x_3	0	0	0	1	0	1
x_4	1	1	1	0	0	1
x_5	1	0	0	1	0	0
x_6	0	0	0	0	0	0

(a) Draw a decision tree of minimal depth that correctly classifies the examples in this dataset.

(b) How much information is needed to classify an example in this case? (Reminder: the amount of information needed to classify an example is $-(p*\log p)-(n*\log n)$, where p is the probability of a positive answer and n is the probability of a negative answer.)

(c) How much information is needed to classify an example *given* that a is 1? What if a is 0?

(d) So, what is the information gain from attribute a? What is the information gain from d given a?