Computer Science Department
Stanford University
Comprehensive Examination in Numerical Analysis
Fall 2002

1. (10 points) Consider a system of linear equations

$$Ax = b \tag{1}$$

where A is a real $n \times n$ non-singular matrix, $b \in R^n$.

a) (3 points) Describe how to use the LU decomposition (assume it is given) to solve the system of linear equations (the Gaussian elimination). Why might one need pivoting?

b) (4 points) Describe how to use the QR decomposition (assume it is given) for solving (1). In what cases might one use the QR decomposition rather than the LU decomposition?

c) (3 points) Describe an iterative method to solve (1) in case the matrix A is a symmetric positive definite and discuss its convergence properties. In what cases would iterative methods prevail over the direct ones?

2. (10 points) Consider a system of ordinary differential equations

$$\begin{pmatrix} x \\ y \end{pmatrix}_t = A \begin{pmatrix} x \\ y \end{pmatrix}, \quad \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}, \quad A \in R^{2x2}. \tag{2}$$

a) (3 points) Assume A has a complete set of eigenvectors. How do eigenvalues influence stability (definition of stability: $\left|\begin{matrix} x(t) \\ y(t) \end{matrix}\right| \le K \left|\begin{matrix} \varphi_1 \\ \varphi_2 \end{matrix}\right|$ for $t \ge 0$, where K is independent of time and initial data) of this problem?

b) (2 points) Assume A has only one eigenvalue with algebraic multiplicity 2. Do the stability conditions remain the same?

c) (5 points) Let $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Describe how to solve this system with the forward Euler scheme. Is it a good numerical method to solve this problem? If not, which method would you use instead?
Hint: Solve (2) and consider the analytical stability of its solution.

3. (10 points) Numerical Integration.

a) (3 points) Write the formula for the composite trapezoidal rule scheme.

b) (2 points) Define the degree of accuracy for a numerical quadrature scheme.

c) (5 points) Derive Simpson's method using a Taylor polynomial and in the process show that it has degree of accuracy 3.