

## 2001 Comprehensive Examination Artificial Intelligence

**1. Search.** (20 points) Consider a search tree with uniform branching factor  $b$  and depth  $d$ , and consider a search problem for which there is a single solution in the tree at depth  $k$ . A solution at the root of the tree is depth 0. Give expressions for the worst case cost of finding the solution, in terms of nodes visited, for (a) breadth-first search, (b) depth-first search, and (c) iterative deepening (starting at depth 0 and incrementing by 1 on each iteration). Give closed form expressions, if you can; but sums are okay. If you are unable to do this problem in general, you can still get some points by answering the question for the special case of  $b=2$ . And, if that is still too daunting, you may be able to scrape out a point or two by fixing  $k$  and  $d$  as well.

**2. Automated Reasoning.** (30 points) Two questions related to resolution.

(a) Consider the following pairs of expressions.  $u, v, w, x, y, z$  are variables; all other letters are constants. For each pair, say whether or not they are unifiable; if the answer is yes, give the most general unifier.

$$p(x,b) \text{ and } p(f(y,y),y)$$

$$q(x,f(y,a),g(g(x))) \text{ and } q(z,f(z,u),y)$$

(b) Given the following premises, use the resolution method to prove  $\neg p(c,a)$ .

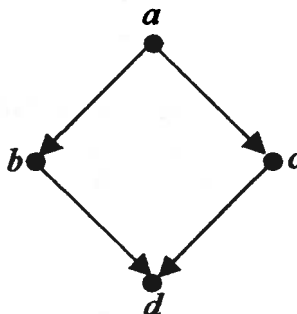
$$\forall y. \forall z. (p(y,z) \Rightarrow \neg p(z,y))$$

$$\forall x. (p(b,x) \Rightarrow p(a,x))$$

$$p(b,c) \vee p(a,c)$$

Note that this is a question about the resolution method. You will get zero points for proving it in any other way.

**3. Probability.** (30 points) Adapted from Nilsson's *Artificial Intelligence: A New Synthesis*. The admissions committee for a major university wants to know the probability that an applicant is qualified given that the person is admitted. It has the belief network shown below.



$$\begin{aligned}
p(a) &= 0.5 \\
p(b|a) &= 1 \\
p(b|\neg a) &= 0.5 \\
p(c|a) &= 1 \\
p(c|\neg a) &= 0.5 \\
p(d|b, c) &= 1 \\
p(d|b, \neg c) &= 0.5 \\
p(d|\neg b, c) &= 0.5 \\
p(d|\neg b, \neg c) &= 0
\end{aligned}$$

*a* - applicant is qualified  
*b* - applicant has a high grade point average  
*c* - applicant has a high graduate record examination score  
*d* - applicant is admitted

What is the probability that an admitted student is qualified? In other words, calculate  $p(a|d)$ .

**4. Natural Language.** (20 points) Consider the augmented phrase structure grammar shown below.

$$\begin{aligned}
S(r(x, z) \wedge r(y, z)) &\rightarrow Q(r(\text{both}(x, y), z)) \\
Q(w(u, v)) &\rightarrow NP(u) \text{ Verb}(w) NP(v) \\
NP(x) &\rightarrow \text{Noun}(x) \\
NP(\text{both}(x, y)) &\rightarrow NP(x) \text{ and } NP(y) \\
\text{Noun}(\text{tom}) &\rightarrow \text{Tom} \\
\text{Noun}(\text{dick}) &\rightarrow \text{Dick} \\
\text{Noun}(\text{harry}) &\rightarrow \text{Harry} \\
\text{Noun}(\text{mary}) &\rightarrow \text{Mary} \\
\text{Verb}(\text{hates}) &\rightarrow \text{hate} \\
\text{Verb}(\text{hates}) &\rightarrow \text{hates}
\end{aligned}$$

(a) Given that *s* is the top-level non-terminal, is there a semantic interpretation for the expression *Mary hates Tom and Harry*? If so, what is it?

(b) Given that *s* is the top-level non-terminal, is there a semantic interpretation for the expression *Tom and Harry hate Mary*? If so, what is it?

(c) Change the augmentations on the existing rules to eliminate ungrammatical sentences like *Tom and Harry hates Mary* (without eliminating the corresponding grammatical sentences). If you are unable to do this, you can still get partial credit by changing the rules themselves.