SOLUTIONS

2001 Comprehensive Examination
Artificial Intelligence

1. Search. If we assume that the choices at each branch point are searched left to right, then the worst case occurs when the solution lies at depth k down the rightmost branch of the tree.

In breadth-first search, the tree is searched only to the level at which the solution lies. This leads to the following expression.

$$\frac{b^{k+1}-1}{b-1}$$

In depth-first search, each branch is explored to its greatest depth before backing up. We search the entire tree except for the nodes lying below the solution. This leads to the following expression.

$$\frac{b^{d+1}}{b-1}-\frac{b^{d-k+1}}{b-1}+1$$

Each iteration in iterative deepening search repeats the work of the previous iteration. The following expression gives the worst-case cost.

$$\sum_{i=0}^{k}\frac{b^{i+1}-1}{b-1}$$

The good news is that, like breadth-first search, it does not need to search to greater depth than the solution. Also, since the tree is growing exponentially, this repeated work is always bounded by the amount of work in searching the next level; so the increase in cost over the other methods is not very great. In the case of a uniform tree, iterative deepening searches no more that $b/(b-1)$ worse than breadth-first search in terms of nodes searched.

2. Automated Reasoning.

(a) $\{x \leftarrow f(b,b), y \leftarrow b)\}$

(b) Not unifiable. Applying the unification algorithm up to the last argument of each expression, we get the substitution $\{x \leftarrow z, y \leftarrow z, u \leftarrow a\}$ and the clauses $h(z,f(z,a),g(g(z))$, and $h(z,f(z,a),z)$. At this point, the occurs check prevents us from binding z to $g(g(z))$.

(b) In resolution we first add the negated goal to the premises. In this case, the goal $\neg p(c,a)$ becomes $p(c,a)$ after negation. We then convert the premises and negated goal to clausal form, leading to the following clauses.

$$\{\neg p(y,z), \neg p(z,y)\}$$
$$\{\neg p(b,x), p(a,x)\}$$
$$\{p(b,c), p(a,c)\}$$
$$\{p(c,a)\}$$

We then apply the resolution rule to these clauses to produce the empty clause.

1.	$\{\neg p(y,z), \neg p(z,y)\}$	Premise
2.	$\{\neg p(b,x), p(a,x)\}$	Premise
3.	$\{p(b,c), p(a,c)\}$	Premise
4.	$\{p(c,a)\}$	Negated Goal
5.	$\{p(a,c)\}$	2,3
6.	$\{\neg p(c,a)\}$	1,5
7.	$\{\}$	4,6

3. Probability. One way to solve the problem is shown below.

$$p(a \mid d) = \frac{p(a,d)}{p(d)}$$

$$p(a,d) = p(a,b,c,d) + p(a,b,\neg c,d) + p(a,\neg b,c,d) + p(a,\neg b,\neg c,d)$$

$$p(\neg a,d) = p(\neg a,b,c,d) + p(\neg a,b,\neg c,d) + p(\neg a,\neg b,c,d) + p(\neg a,\neg b,\neg c,d)$$

$$p(d) = p(a,d) + p(\neg a,d)$$

Using the given probabilities, we can compute the following values.

$$p(a,b,c,d) = 0.5$$
$$p(a,b,\neg c,d) = 0$$
$$p(a,\neg b,c,d) = 0$$
$$p(a,\neg b,\neg c,d) = 0$$
$$p(\neg a,b,c,d) = 0.125$$
$$p(\neg a,b,\neg c,d) = 0.0675$$
$$p(\neg a,\neg b,c,d) = 0.0675$$
$$p(\neg a,\neg b,\neg c,d) = 0$$

Using these values and the equations above, we get the following results.

$$p(a,d) = 0.5$$

$$p(d) = 0.75$$

$$p(a \mid d) = 2/3$$

Note that one can take a short cut in this case by noticing that $p(d|a)=1$, i.e. all qualified applicants are admitted (though some non-qualified applicants are also admitted). From this observation, one can directly compute $p(a,d)=0.5$ using the definition of conditional probability.

4. Natural Language.

(a) There is none.

(b) *hates(tom,mary) & hates(harry,mary)*

(c) One way is to add a number parameter, as shown below.

$S(r(x, z) \wedge r(y, z)) \rightarrow Q(r(both(x,y), z))$
$Q(w(u, v)) \rightarrow NP(u,n)\ Verb(w,n)\ NP(v)$
$NP(x,s) \rightarrow Noun(x)$
$NP(both(x, y),p) \rightarrow NP(x)$ **and** $NP(y)$
$Noun(tom) \rightarrow$ **Tom**
$Noun(dick) \rightarrow$ **Dick**
$Noun(harry) \rightarrow$ **Harry**
$Noun(mary) \rightarrow$ **Mary**
$Verb(hates,p) \rightarrow$ **hate**
$Verb(hates,s) \rightarrow$ **hates**

It is also possible to accomplish this by splitting the rules for *qs*, *np*, and *verb*; but this can be more cumbersome.