Comprehensive Examination in LOGIC

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READ THIS FIRST!

1. You should mark your answers only in the ANSWER SHEET that is provided with this part of the Comprehensive Examination. Be sure to write your MAGIC NUMBER on the answer sheet.
2. This exam is OPEN BOOK.
3. All questions in the exam are Yes/No questions, grouped in fours. Write "Y" in the boxes of those choices that you think are correct. Write "N" in the boxes of those choices that you think are not correct.
4. You will receive **1 point** for each box you fill out correctly. **1/2 points** will be deducted for each box that is filled out incorrectly. No points will be deducted for boxes that are left blank.
5. The exam takes 60 minutes and there are 13 questions (with four choices each).

x 50

1. Consider two *arbitrary* predicate logic sentences $\mathcal{F}$ and $\mathcal{G}$. Which of the following statements are correct?

(A) If $(\forall *)\mathcal{F}$ has no falsifying interpretation then $(\exists *)\neg\mathcal{F}$ is not satisfiable.

(B) If $(\forall *)\mathcal{F}$ is not valid, then $(\exists *)[\textit{if } \mathcal{F} \textit{ then } \mathcal{G}]$ is satisfiable.

(C) If $(\forall *)\mathcal{F}$ has a falsifying interpretation, then $(\exists *)\mathcal{F}$ is not valid.

(D) $(\forall x)\mathcal{F}[x]$ is valid
if and only if
both $(\forall x)(\mathcal{F}[x] \equiv \mathcal{F}[a])$ and $(\exists x)\mathcal{F}[x]$ are valid.

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2. Decide for the following pairs of sentences $\mathcal{F}, \mathcal{G}$, whether $\mathcal{G}$ is a correct skolemization of $\mathcal{F}$ (i.e., whether $(\exists *)\mathcal{G}$ is valid precisely when $\mathcal{F}$ is valid). x, y, z are variables, a, b are constants.

(A) $\mathcal{F}: \ (\exists x)\ [(\textit{not}\ \ ((\exists y)\ p(x,y)))\ \textit{and}\ q(x)]$
$\mathcal{G}: \ (\textit{not}\ p(x,a))\ \textit{and}\ q(x)$

(B) $\mathcal{F}: \ [(\forall x)\ p(x)] \ \equiv \ [(\forall y)\ q(y)]$
$\mathcal{G}: \ [\textit{if}\ p(x)\ \textit{then}\ q(a)]\ \textit{and}\ [\textit{if}\ q(y)\ \textit{then}\ p(b)]$

Which of the following sentences are valid?

(C) $$\begin{bmatrix} P \textit{ and} \\ \textit{if } Q \textit{ then } R \end{bmatrix} \equiv \begin{bmatrix} \textit{if } Q \\ \textit{then } (P \textit{ and } R) \end{bmatrix}$$

(D) $[\textit{if}\ (P \equiv Q)\ \textit{then}\ P]\ \ \textit{or}\ \ [\textit{if}\ Q\ \textit{then}\ (P \equiv Q)]$

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3. Suppose that starting from an initial tableau, by correctly applying the deductive tableau system for predicate logic, one arrives at tableau $\mathcal{T}$. In which of the following cases can we conclude that the initial tableau is valid?

(A) There are no goals in $\mathcal{T}$.

(B) One of the goals in $\mathcal{T}$ is *false*.

(C) One of the goals in $\mathcal{T}$ is *true*.

(D) One of the assertions in $\mathcal{T}$ is unifiable with one of the goals.

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4. Consider the theory Γ, defined by the following set of axioms $\mathcal{A}$ ($c_1, c_2, c_3, \ldots$ are constants, v is a variable, p a predicate symbol):

$$\mathcal{A} = \{ \quad \textit{not}\ [(\forall v)\ p(v)], \\ p(c_1),\ p(c_2),\ p(c_3),\ \ldots \}$$

Which of the following statements are true?

(A) Γ is consistent.

(B) Γ is satisfiable.

(C) Γ has a model with finite domain.

(D) The sentence $(\exists x)\ p(x)$ is valid in Γ.

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5. In the following deductive tableau x, y and z are variables, a is a constant.

	assertions	goals
A1	*if* $(p(x,y)$ *or* $q(a,a))$ *then* $q(y,y)$	
G2		$q(a,z)$ *or* $r(z)$

Which of the following can appear as the result of an application of one of the *resolution* rules?

(A)

A3	$($*not* $p(x,a))$ *or* $q(y,y)$	

(B)

G3		$r(a)$ *and* $p(x,a)$

(C)

G3		$r(a)$ *and* $($*not* $q(y,y))$

(D)

A3	*false*	

6. Let $\mathcal{A}$ be an arbitrary set of axioms defining a theory and let $\mathcal{F}$ and $\mathcal{G}$ be two arbitary closed sentences of predicate logic. Which of the following statements are correct?

   (A) If $\mathcal{F}$ implies $\mathcal{G}$, and $\mathcal{G}$ is not valid in the theory defined by $\mathcal{A}$, then $\mathcal{F}$ is not valid in the theory defined by $\mathcal{A}$

   (B) If *true* is the only axiom in $\mathcal{A}$, then there are no valid sentences in the theory defined by $\mathcal{A}$

   (C) If *false* is the only axiom in $\mathcal{A}$, then there are no valid sentences in the theory defined by $\mathcal{A}$

   (D) If $\mathcal{F}$ is not valid in the theory defined by $\mathcal{A}$, then $(not\ \mathcal{F})$ is valid in $\mathcal{A}$.

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7. Consider the following statements about well-founded relations. Which are correct?

   (A) If a binary relation R over a *finite* set $\mathcal{S}$ is both irreflexive and transitive, then R is well-founded over $\mathcal{S}$.

   (B) The less-than-or-equal relation $\leq$ in the theory of the nonnegative integers is well-founded.

   (C) Let $\prec_1$ and $\prec_2$ be two well-founded relations over a set $\mathcal{S}$. Then the relation $\prec$ with $x \prec y \equiv (x \prec_1 y\ or\ x \prec_2 y)$ is well-founded over $\mathcal{S}$.

   (D) Every well-founded relation is transitive.

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8. Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be arbitrary theories, defined by two sets of axioms: $\mathcal{A}_1$ and $\mathcal{A}_2$, respectively. Now consider theory $\mathcal{T}_3$, defined by the union $\mathcal{A}_1 \cup \mathcal{A}_2$: Which of the following statements are correct?

(A) If $\mathcal{F}$ is valid in $\mathcal{T}_3$,
then it is valid in $\mathcal{T}_1$ or in $\mathcal{T}_2$.

(B) If $\mathcal{F}$ is valid in theory $\mathcal{T}_1$ and
$\mathcal{G}$ is valid in theory $\mathcal{T}_2$,
then ($\mathcal{F}$ *and* $\mathcal{G}$) is valid in theory $\mathcal{T}_3$.

(C) Assume that $\mathcal{T}_1$ and $\mathcal{T}_2$ are both complete.
Suppose additionally that $\mathcal{T}_3$ is satisfiable.
Then $\mathcal{T}_1$ and $\mathcal{T}_2$ have the same set of consequences.

(D) Assume that $\mathcal{T}_1$ is complete.
Suppose additionally that $\mathcal{T}_3$ is satisfiable.
Then $\mathcal{T}_3$ is decidable.

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9. Which of the following statements about *unification* and *substitution* are correct?

(A) $\{x \leftarrow f(x), y \leftarrow a\}$ is a most general unifier of
$p(x, a)$ and $p(f(x), y)$

(B) The following substitutions are equally general:
$\{x \leftarrow y,\ y \leftarrow z,\ z \leftarrow x\}$, $\{x \leftarrow z,\ y \leftarrow x,\ z \leftarrow y\}$, $\{\}$

(C) For three terms t_1, t_2 and t_3,
if t_1 and t_2 are unifiable, and t_2 and t_3 are unifiable,
then t_1 and t_3 are unifiable.

(D) For substitutions $\theta_1, \theta_2, \theta_3$,
if θ_2 is more general than θ_1 and θ_3 is more general than θ_2,
then θ_3 is more general than θ_1.

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10. Consider the following deductive tableau $\mathcal{T}$:

	assertions	goals
G1		$(\forall x)$ [*if* $p(x)$ *then* $q(y)$]

In each of the following cases, consider the new tableau that is constructed by adding the given new row to $\mathcal{T}$.
In which cases is the new tableau equivalent to the original tableau $\mathcal{T}$?
(x, y are variables)

(A)

A2	$p(x)$	

(B)

A2	$q(y)$	

(C)

G2		$q(y)$

(D)

A2	*true*	

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11. Let $\mathcal{T}$ be a theory defined by a set of axioms $\mathcal{A}$ and let $\mathcal{F}$ be a closed sentence of predicate logic that is *not* valid in $\mathcal{F}$. Now consider $\mathcal{T}'$, the theory defined by the axioms $\mathcal{A}' = \mathcal{A} \cup \{\mathcal{F}\}$. Which of the following statements are correct?

(A) The axioms $\mathcal{A}'$ must be independent.

(B) $\mathcal{T}'$ must be consistent.

(C) $\mathcal{T}'$ must be inconsistent.

(D) Let $Mod(U)$ denote the collection of models of a theory U. Then $Mod(\mathcal{T}') \subseteq Mod(\mathcal{T})$.

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12. In which of the following cases can the described algorithm exist?

(A) Given any sentence of propositional logic, the algorithm returns `true` if the sentence is satisfiable, and returns `false` otherwise.

(B) Given any sentence of predicate logic, the algorithm returns `true` if the sentence is satisfiable, and returns `false` otherwise.

(C) Given any sentence of predicate logic, the algorithm returns `true` precisely when the sentence is satisfiable, and either returns `false` or does not terminate when the sentence is not satisfiable.

(D) Given any sentence of predicate logic, the algorithm returns `false` precisely when the sentence is not satisfiable, and either returns `true` or does not terminate when the sentence is satisfiable.

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13. Which of the following statements are true? ☺

✓ ✓

(A) Choice **(B)** is false. T F

(B) Choice **(C)** is true. F T

(C) Choice **(A)** is false. F T

(D) Fewer than two of the four choices are true. F

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