Comprehensive Examination in LOGIC

Computer Science Department
Stanford University

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READ THIS FIRST!

1. You should mark your answers only in the ANSWER SHEET that is provided with this part of the Comprehensive Examination. Be sure to write your MAGIC NUMBER on the answer sheet.

2. This exam is OPEN BOOK.

3. This exam is multiple choice. Mark "**X**" in at most one box for each question. Occasionally, the last option will be "**(E)** none of the above". Mark this choice if none of **(A)**, **(B)**, **(C)** and **(D)** are correct.

4. The exam takes 60 minutes and there are 18 questions in the exam. You will receive **1** point for each CORRECT answer, **0** points for not answering, and **-1/3** points for each WRONG answer.

55

1. Consider the following sentences of propositional logic

$$\mathcal{S}_1: \ (\textit{if } P \textit{ then } Q) \equiv (\textit{if } Q \textit{ then } P)$$

$$\mathcal{S}_2: \ P \equiv Q$$

Which of the following statements is correct?

(A) $\mathcal{S}_1$ is valid, $\mathcal{S}_2$ is satisfiable but not valid
(B) $\mathcal{S}_2$ is valid, $\mathcal{S}_1$ is satisfiable but not valid
(C) $\mathcal{S}_1$ is not valid, $\mathcal{S}_1$ implies $\mathcal{S}_2$, $\mathcal{S}_2$ does not imply $\mathcal{S}_1$
(D) $\mathcal{S}_2$ is not valid, $\mathcal{S}_2$ implies $\mathcal{S}_1$, $\mathcal{S}_1$ does not imply $\mathcal{S}_2$
(E) $\mathcal{S}_1$ and $\mathcal{S}_2$ are equivalent.

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2. Let $\mathcal{F}, \mathcal{G}$ stand for arbitrary sentences of propositional logic. What is the relationship between these three statements?

(a) $\mathcal{F}$ is equivalent to $\mathcal{G}$.

(b) $(\mathcal{F} \equiv \mathcal{G})$ is valid.

(c) $\mathcal{F}$ is valid precisely when $\mathcal{G}$ is valid.

(A) (a), (b) and (c) are equivalent.
(B) (a), (b) and (c) are not all equivalent, but (a) implies (b), (b) implies (c)
(C) (a), (b) and (c) are not all equivalent, but (b) implies (c), (c) implies (a)
(D) (a), (b) and (c) are not all equivalent, but (c) implies (a), (a) implies (b)
(E) none of the above

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3. Which of the following is a correct proof of validity by the method of falsification?

(A) $\begin{array}{lllllllllll} (\textit{not} & (\textit{if} & P & \textit{then} & Q & \textit{and} & R)) & \textit{and} & (R & \textit{and} & Q) \\ T & F & T & & T & \substack{T\\F} & T & T & T & T & T \end{array}$

(B) $\begin{array}{llllll} \textit{if} & (R & \textit{and} & Q) & \textit{then} & Q \\ F & \substack{T\\F} & T & F & & F \end{array}$

(C) $\begin{array}{lllllllllll} (\textit{not} & (P & \textit{or} & Q)) & \textit{and} & (\textit{if} & Q & \textit{then} & (P & \textit{or} & Q\ )) \\ F & T & \substack{T\\F} & T & T & F & F & & F & F & F \end{array}$

(D) $\begin{array}{llllllllll} \textit{if} & (P & \equiv & Q) & \textit{then} & ((\textit{if} & P & \textit{then} & Q) & \textit{or} & R) \\ F & T & T & \substack{T\\F} & & F & T & & F & F & F \end{array}$

(E) $\begin{array}{llllllllll} \textit{if} & (P & \textit{and} & Q) & \textit{then} & ((\textit{if} & Q & \textit{then} & P) & \textit{or} & P) \\ F & T & T & T & & F & T & & F & F & F \end{array}$

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4. Consider the following theories:

   (a) number theory, with zero (0) and successor (S)

   (b) number theory, with 0, S and addition (+).

   (c) number theory, with 0, S, + and multiplication ($\cdot$).

   For which of them does an algorithm A exist such that given an arbitrary sentence S, A halts if S is valid and does not halt otherwise.

   (A) only (a)
   (B) only (b)
   (C) only (a) and (b)
   (D) (a), (b) and (c)
   (E) none of the above

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5. Which of the following sentences of propositional logic are instances of the polarity proposition?

$$\mathcal{S}_1: \quad \textit{if } (\textit{if } p \textit{ then } q) \\ \textit{then } [\textit{if } (q \textit{ and } (\textit{if } p \textit{ then } r)) \textit{ then } (q \textit{ and } (\textit{if } q \textit{ then } r))]$$

$$\mathcal{S}_2: \quad \textit{if } (\textit{if } q \textit{ then } p) \\ \textit{then } [\textit{if } (q \textit{ and } (\textit{if } p \textit{ then } r)) \textit{ then } (q \textit{ and } (\textit{if } q \textit{ then } r))]$$

$$\mathcal{S}_3: \quad \textit{if } (\textit{if } p \textit{ then } q) \\ \textit{then } (\textit{if } q \textit{ then } p)$$

   (A) only $\mathcal{S}_1$
   (B) only $\mathcal{S}_2$
   (C) only $\mathcal{S}_3$
   (D) only $\mathcal{S}_2$ and $\mathcal{S}_3$
   (E) $\mathcal{S}_1$, $\mathcal{S}_2$, and $\mathcal{S}_3$.

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6. In the following deductive tableau x, y and z are variables, a is a constant.

	assertions	goals
G1		$p(a, x)$ *or* $q(z)$
A2	*if* $r(w, y)$ *then* $p(y, y)$	

Which of the following is the result of a AG-resolution of A2 and G1?

(A)

G3		$r(w, a)$

(B)

G3		$r(w, y)$

(C)

G3		*false*

(D)

G3		*not* $r(w, a)$

(E)

A3	$r(w, y)$	

7. Let $\mathcal{T} = \langle \mathcal{A}, \mathcal{G} \rangle$ be a deductive tableau in propositional logic in which $\mathcal{A} = \{A_1, \ldots, A_n\}$ are the assertions and $\mathcal{G} = \{G_1, \ldots, G_n\}$ are the goals.
Which of the following statements are correct?

(a) $\mathcal{T}$ is true under an interpretation I if and only if the sentence

$$\textit{if } (A_1 \textit{ and } \ldots \textit{ and } A_n) \textit{ then } (G_1 \textit{ and } \ldots \textit{ and } G_n)$$

is true under I.

(b) If $\mathcal{A} = \mathcal{A}' \cup \{A\}$, then $\mathcal{T}$ is equivalent to $\mathcal{T}' = \langle \mathcal{A}', \mathcal{G} \cup \{\textit{not } A\} \rangle$.

(c) For two tableaux $\mathcal{T}_1 = \langle \mathcal{A}_1, \mathcal{G}_1 \rangle$ and $\mathcal{T}_2 = \langle \mathcal{A}_2, \mathcal{G}_2 \rangle$, with

$$\mathcal{A}_1 \subseteq \mathcal{A}_2 \subseteq \mathcal{A} \quad \text{and} \quad \mathcal{G}_1 \subseteq \mathcal{G}_2 \subseteq \mathcal{G}$$

if $\mathcal{T}_1$ is equivalent to $\mathcal{T}$ then
$\mathcal{T}_1$ is equivalent to $\mathcal{T}_2$ and $\mathcal{T}_2$ is equivalent to $\mathcal{T}$.

(A) only (a)
(B) only (b)
(C) (a), (b), and (c)
(D) only (b) and (c)
(E) none of the above

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8. Let $\mathcal{F}$ be an arbitrary sentence of predicate logic, and $\mathcal{T}$ an arbitrary finitely axiomatizable theory. Assume you have a resolution theorem prover that follows a resolution strategy $\mathcal{S}$. You give the prover the axioms of $\mathcal{T}$ and try to prove $\mathcal{F}$. Five weeks later the prover reports that no new resolvents can be produced and no proof has been found. Which of the following statements must be correct?

(A) if $\mathcal{T}$ is complete, then $\mathcal{T}$ contains the sentence *not* $\mathcal{F}$.
(B) either $\mathcal{S}$ is incomplete or $\mathcal{T}$ is incomplete.
(C) if $\mathcal{S}$ is complete and $\mathcal{T}$ is complete, then $\mathcal{T}$ contains the sentence *not* $\mathcal{F}$.
(D) $\mathcal{F}$ is valid under $\mathcal{T}$.
(E) none of the above

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9. Let $\mathcal{F}[x]$ and $\mathcal{G}[y]$ be arbitrary predicate logic sentences such that x is the only free variable in $\mathcal{F}[x]$ and y is the only free variable in $\mathcal{G}[y]$. Let a be a constant not appearing in $\mathcal{F}[x]$ or $\mathcal{G}[y]$. Consider the following sentences:

$$\begin{aligned} \mathcal{S}_1 &: ((\forall x)\ \mathcal{F}[x])\ \textit{and}\ ((\exists y)\ \mathcal{G}[y]) \\ \mathcal{S}_2 &: \mathcal{F}[a]\ \textit{and}\ \mathcal{G}[y] \end{aligned}$$

Which of the following statements is correct?

(a) if $\mathcal{S}_1$ is valid then $(\exists y)\mathcal{S}_2$ is valid

(b) if $(\exists y)\mathcal{S}_2$ is valid then $\mathcal{S}_1$ is valid

(c) if $\mathcal{S}_1$ is satisfiable then $(\forall y)\mathcal{S}_2$ is satisfiable

(d) if $(\forall y)\mathcal{S}_2$ is satisfiable then $\mathcal{S}_1$ is satisfiable

(A) (a) only
(B) (a) and (b) only
(C) (c) only
(D) (c) and (d) only
(E) none of the above

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10. Let t_1, t_2 and t_3 be terms such that θ_1 is a most general unifier (m.g.u.) of t_1 and t_2 and θ_2 is a m.g.u. of t_2 and t_3. Let $\theta = \theta_1 \Box \theta_2$ be the composition of θ_1 and θ_2. Which of the following statements is correct?

(A) θ must be a m.g.u. of t_1,t_2, and t_3
(B) θ must be a unifier of t_1,t_2, and t_3, but not a m.g.u.
(C) θ must be a m.g.u. of t_1 and t_3
(D) θ must be a unifier of t_1 and t_3, but not a m.g.u.
(E) none of the above

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11. Consider three different implementations, (a), (b) and (c), of a Deductive Tableau system, each of which is derived from a sound and complete implementation by a single modification to the unification procedure. For each implementation, the only change made is

(a) for x and y, the unification procedure returns the substitution $\{x \mapsto z, y \mapsto z\}$, where z is a new variable

(b) for $f(x, y)$ and $f(g(z), a)$, the procedure returns $\{x \mapsto g(a), y \mapsto a, z \mapsto a\}$

(c) for $f(x, y)$ and $f(g(y), x)$, the procedure returns $\{x \mapsto g(z), y \mapsto g(z)\}$, where z is a new variable

(In each case, x, y, and z are variables, and a is a constant).
Which of the following statements is correct?

(A) implementation (a) is sound, (b) is complete, (c) is not sound
(B) implementation (a) is complete, (b) is sound, (c) is not sound
(C) implementation (a) is sound and complete, (b) is sound, (c) is sound
(D) implementation (a) is complete, (b) is sound and complete, (c) is not sound
(E) none of the above

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12. Let $\mathcal{F}$ be an arbitrary valid sentence of predicate logic with equality. A deductive tableau proof that $\mathcal{F}$ is valid can be obtained by:

(a) starting with only the goal F and using only the resolution and equality rules

(b) starting with only the goal F and a finite number of axioms of equality, and using only the resolution rule

(c) starting with only the goal F and a finite number of axioms of equality, and using only the resolution and equality rules

Which of the following statements is correct?

(A) only (c) is true
(B) only (b) and (c) are true
(C) only (a) and (c) are true
(D) (a), (b) and (c) are true
(E) none of the above

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13. Let Σ be a finite set of sentences of propositional logic. Which of the following statements are correct?

(a) There is an effective procedure which, given a propositional sentence $\mathcal{S}$, will decide whether or not $\Sigma \models \mathcal{S}$.

(b) There is an effective procedure which enumerates all interpretations that satisfy all sentences in Σ.

(c) There is an effective procedure which enumerates the set of consequences of Σ.

(A) (a), (b), and (c)
(B) only (a) and (b)
(C) only (a) and (c)
(D) only (b) and (c)
(E) none of the above

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14. Which of the following statements are true? A set of sentences $\{S_1, S_2, \ldots, S_n\}$ is consistent if and only if

(a) the sentence S_1 *or* S_2 *or* ... *or* S_n is valid.

(b) the sentence S_1 *and* S_2 *and* ... *and* S_n is satisfiable.

(c) the sentence (*not* S_1) *or* (*not* S_2) *or* ... *or* (*not* S_n) is not valid.

(A) (a), (b), and (c)
(B) only (a) and (b)
(C) only (b)
(D) only (b) and (c)
(E) none of the above

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15. Consider the following terms:

$$\begin{aligned} t_1 &: \ f(x, g(x)) \\ t_2 &: \ f(y, g(y)) \\ t_3 &: \ f(g(z), g(y)) \end{aligned}$$

Which of the following statements are correct?

(a) t_1 and t_2 are unifiable

(b) t_2 and t_3 are unifiable

(c) t_1 and t_3 are unifiable

(d) the tuple (t_1, t_2, t_3) is unifiable

(A) only (a)
(B) only (a) and (b)
(C) only (a) and (b) and (c)
(D) (a), (b), (c) and (d)
(E) none of the above

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16. Let Σ_1 be a set of sentences of predicate logic and S a sentence of predicate logic such that $\Sigma_1 \models S$. Which of the following statements are correct?

(a) For every finite subset $\Sigma_2 \subseteq \Sigma_1$, $\Sigma_2 \models S$.

(b) If every finite subset $\Sigma_2 \subseteq \Sigma_1$ is satisfiable then $\Sigma_1 \cup \{S\}$ is satisfiable

(c) If Σ_1 is satisfiable, then, for every finite subset $\Sigma_2 \subseteq \Sigma_1$, the set $\Sigma_2 \cup \{S\}$ has a model.

(A) (a), (b), and (c)
(B) only (a) and (c)
(C) only (b) and (c)
(D) only (c)
(E) none of the above

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17. Let $\mathcal{F}, \mathcal{G}$ be arbitrary predicate logic sentences, and $\mathcal{S}_1$, $\mathcal{S}_2$ be defined as follows:

$$\begin{aligned} \mathcal{S}_1 &: (\forall x)(\textit{if } \mathcal{F} \textit{ then } \mathcal{G}) \\ \mathcal{S}_2 &: \textit{if } (\forall x)\mathcal{F} \textit{ then } (\forall x)\mathcal{G} \end{aligned}$$

Which of the following statements are true?

(a) $\mathcal{S}_1$ implies $\mathcal{S}_2$.

(b) assuming $\mathcal{S}_1$ and $\mathcal{S}_2$ are closed, then if $\mathcal{S}_1$ is valid, $\mathcal{S}_2$ is also valid.

(c) assuming $\mathcal{S}_1$ and $\mathcal{S}_2$ are closed, then if $\mathcal{S}_2$ is valid, $\mathcal{S}_1$ is also valid.

(A) only (a)
(B) only (b)
(C) only (a) and (b)
(D) (a) and (b) and (c)
(E) none of the above

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18. Which of the following equivalences are valid?

(a)
$$\begin{gathered} (P \textit{ and } (\textit{not } Q)) \textit{ or } (Q \textit{ and } (\textit{not } R)) \textit{ or } (R \textit{ and } (\textit{not } P)) \\ \equiv \\ ((\textit{not } P) \textit{ and } Q) \textit{ or } ((\textit{not } Q) \textit{ and } R) \textit{ or } ((\textit{not } R) \textit{ and } P) \end{gathered}$$

(b)
$$\begin{gathered} \textit{if } (\textit{if } P \textit{ then } P) \textit{ then } P \\ \equiv \\ \textit{if } P \textit{ then } (\textit{if } P \textit{ then } P) \end{gathered}$$

(c) $((P \equiv Q) \equiv R) \;\equiv\; (P \equiv (Q \equiv R))$

(d)
$$\begin{gathered} \textit{if } P \textit{ then } (Q \equiv R) \textit{ else } ((\textit{not } Q) \equiv R) \\ \equiv \\ \textit{if } Q \textit{ then } (P \equiv R) \textit{ else } ((\textit{not } P) \equiv R) \end{gathered}$$

(A) (a) only
(B) (b) and (c) only
(C) (d) only
(D) (a), (c), (d) only
(E) none of the above
