Computer Science Department
Stanford University
Comprehensive Examination in Automata and Formal Languages
Autumn 1998

October 26, 1998

READ THIS FIRST!

1. You should write your answers for this part of the Comprehensive Examination in BLUE BOOKS. There are five problems in the exam. Be sure to write your MAGIC NUMBER on the cover of every blue book that you use.

2. The number of POINTS for each problem indicates how elaborate an answer is expected. The exam takes 1 hour.

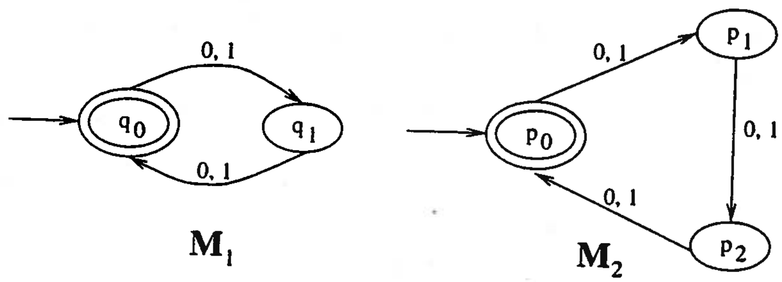
3. This exam is OPEN BOOK. You may use notes, articles, or books—but no help from other sentient agents such as other humans or robots.

4. Show your work, since PARTIAL CREDIT will be given for incomplete answers.

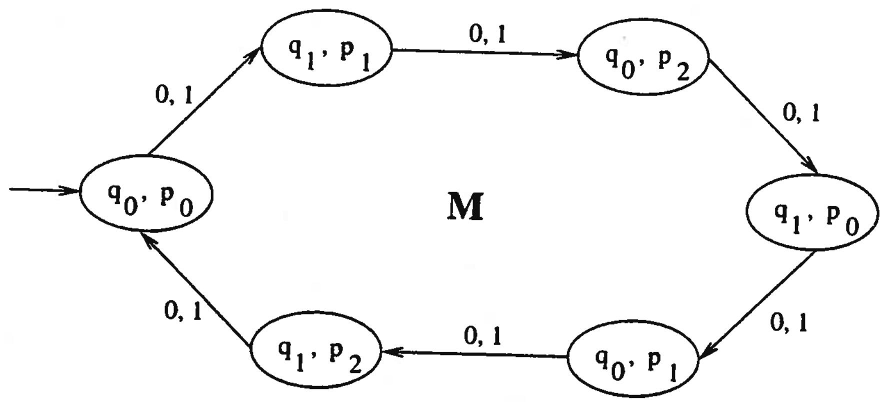
Comprehensive Exam: Autumn 1998-99

Automata and Formal Languages (60 points)

Problem 1. [10 points]
Consider the following DFAs (deterministic finite-state automata) called M_1 and M_2 over the alphabet $\{0,1\}$. Let L_1 be the language of M_1 and L_2 be the language of M_2.



a). [4 points]
Give succinct descriptions of the languages L_1 and L_2.
b). [6 points]
Consider the machine M given below that is the cross-product of the two machines M_1 and M_2. (Running the machine M on an input string corresponds to running M_1 and M_2 together in parallel on that input string. Each state in the machine M corresponds to a pair of states, one each from M_1 and M_2. Each transition in M is the combination of the state transitions from M_1 and M_2.)



For each of the following languages, specify the choice of final states in M that would cause M to accept that language.
(i). $L_1 \cup L_2$
(ii). $L_1 - L_2$ (the set of strings in L_1 that do not belong to L_2)
(iii). $\overline{L_1}$

Problem 2. [10 points]
In the following, R denotes a *regular* language, and C, C' denote *context-free* languages. Classify each of the following statements as being TRUE or FALSE. *You will receive 2 points for each correct answer and −1 point for each incorrect answer.*

a). There must exist a *deterministic* push-down automata that accepts R.
b). There must exist a *deterministic* Turing machine that accepts $C \cap C'$.
c). $R \cap C$ must be regular.
d). $C \cup C'$ cannot be regular.
e). $\overline{C}$ must be recursive.

Problem 3. [20 points]
Classify each of the following languages as being in one of the following classes of languages: *empty, finite, regular, context-free, recursive, recursively enumerable.* You must give the *smallest* class that contains *every possible language* fitting the following definitions. For example, the language of a DFA M could be *empty* or *finite*, and must always be *context-free*, but the smallest class that is appropriate *regular.*

a). $L = \{a^i b^j c^k d^l \mid i = j \text{ or } k = l\}$.
b). $L = \{a^i b^j c^k d^l \mid i = j \text{ and } k = l\}$.
c). $L = \{a^i b^j c^k d^l \mid i = j \text{ and } j = k\}$.
c). $L = \{a^i b^j c^k d^l \mid i = l \text{ and } j = k\}$.
e). $L = \{a^i b^j c^k d^l \mid i \times j \times k \times l \text{ is divisible by } 5\}$.
f). The language of a push-down automaton with only one state.
g). The set of strings encoding all push-down automata that accept non-recursive languages.
h). The language of a PDA with two stacks.
i). The complement of a language in NP.
j). The intersection of a recursive language and a recursively enumerable language.

Problem 4. [10 points]
Suppose we have languages A and B such that A has a *polynomial-time reduction* to B. Classify each of the following statements as being TRUE or FALSE. *You will receive 2 points for each correct answer and −1 point for each incorrect answer.*

a). If B is recursively enumerable, then A must be recursively enumerable.
b). If A is recursive, then B must be recursive.
c). If B is NP-hard, then A must be NP-hard.
d). If A is NP-complete, then B must be NP-complete.
e). It is possible that A is solvable in polynomial time but B is not even in NP.

Problem 5. [10 points]
For any language L define its reversal as follows:

$$L^R = \{w^R \mid w \in L\}.$$

Recall that for a string $w = w_1 w_2 \ldots w_k$, we define its reversal as $w^R = w_n \ldots w_2 w_1$.

Suppose that L is an NP-complete language. Then, is L^R also an NP-complete language? For full credit, you must sketch a proof for your answer.

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