

Automata and Formal Languages (30 points)
Winter 1997/98

Instructions.

1. Write your answers in a blue book. Put your magic number on the cover of every blue book you use.
2. Half an hour is allowed. The questions are worth a total of 30 points.
3. The exam is open book. Notes, articles, and books may be consulted but not people or computers.
4. Show all work. Incomplete answers may receive partial credit.

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1. [10] Prove that $\{a^{\lceil \log_x n \rceil} \mid n \geq 1\}$ is regular, where the base x of the logarithm is real and satisfies $x > 1$. (Here n is integer; when n is real you may find useful that the derivative of $\log_x n$ is $\frac{1}{n \ln x}$.)

2. [10] Assuming that nondeterministic pushdown automata accept exactly the context-free languages, show that nondeterministic *one-state* pushdown automata also accept exactly the context-free languages, provided they accept by empty stack.

3. [10] In this question all languages are over a fixed alphabet Σ . Define a language L to be complete for P (the class of polynomial-time recognizable languages) when every L' in P is polynomial-time reducible to L .

(i) Show that neither the empty language nor Σ^* is complete for P .

(ii) Identify all the other languages in P that are not complete for P .