Automata and Formal Languages (30 points)

Winter 1997/98

SOLUTIONS

1. [10] Prove that $\{a^{\lceil \log_x n \rceil} \mid n \geq 1\}$ is regular, where the base x of the logarithm is real and satisfies $x > 1$. (Here a is a letter of an alphabet and n is integer. When n is real you may find useful that the derivative of $\log_x n$ is $\frac{1}{n \ln x}$.)

Solution. For $n \geq \frac{1}{\ln x}$, $\log_x(n+1) - \log_x n \leq 1$ (consider the slope at n). Hence the language can only omit a^i for $i < \frac{1}{\ln x}$, and the complement of a finite set is regular.

2. [10] Assuming that nondeterministic pushdown automata accept exactly the context-free languages, show that nondeterministic *one-state* pushdown automata also accept exactly the context-free languages, provided they accept by empty stack.

Solution. One-state PDA's accept *at most* the context-free languages because they are a special case of PDA's. The usual proof that every context-free language is accepted by some pushdown automaton constructs a one-state pushdown automaton that accepts by empty stack, whence automata of this kind accept *all* context-free languages.

3. [10] In this question all languages are over a fixed alphabet Σ. Define a language L to be complete for P (the class of polynomial-time recognizable languages) when every L' in P is polynomial-time reducible to L.

(i) Show that neither the empty language nor Σ^* is complete for P.

Solution. There can be no function (polynomial-time computable or not) on Σ^* which sends every member of Σ^* to a member of $\emptyset$ because there is no member of the latter to which to send for example the empty string. The same holds when the two languages are interchanged and "member" is replaced by "nonmember."

(ii) Identify all the other languages in P that are not complete for P.

Solution. The languages in (i) are the only such. Let $u \in L$ and $v \notin L$. Given L' in P, the function $f : \Sigma^* \to \Sigma^*$ sending members of L' to u and nonmembers to v is computable in polynomial time because membership in L' is, and $w \in \Sigma^*$ is in L' if and only if $f(w)$ is in L. Hence L is complete for P.