Computer Science Department
Stanford University
Comprehensive Examination in Automata and Formal Languages
Autumn 1996

October 14, 1996

READ THIS FIRST!

1. You should write your answers for this part of the Comprehensive Examination in BLUE BOOKS. There are three problems in the exam. Be sure to write your MAGIC NUMBER on the cover of every blue book that you use.

2. The number of POINTS for each problem indicates how elaborate an answer is expected. The exam takes 30 minutes.

3. This exam is OPEN BOOK. You may use notes, articles, or books—but no help from other sentient agents such as other humans or robots.

4. Show your work, since PARTIAL CREDIT will be given for incomplete answers.

Automata and Formal Languages (30 points)—Autumn 1996

1. [10] Give context-free grammars generating the following languages over the alphabet $\{a, b\}$. (a) $\{a^i b^j a^{i+j+k} b^k \mid i, j, k \geq 0\}$; (b) All strings with an equal number of a's and b's.

2. [10] Consider the following decision problem. Given a regular expression over the alphabet $\{a, b\}$, decide whether the language it denotes contains a string of palindromes. (A string of palindromes is a member of P^* where P is the set of all palindromes.) State whether this problem is decidable, and briefly sketch the reason.

3. [10] For each of the following sets, state whether it is in NP, coNP, or neither. (If it is both you need not say so, in which case either NP or coNP is a correct answer.) Give a one-sentence reason for each.

(a) The set of three-colorable undirected planar graphs. (A three-coloring of a graph is an assignment of at most three distinct colors to the vertices of the graph such that every edge has different colors at each end.)

(b) The set of finite sets of signed binary integers having no subset adding to zero.

(c) (The knapsack problem). The set of pairs (w, c) where w is a finite list of positive binary numbers (items available to go in the knapsack) and c is an integer (the size of the knapsack) such that some subset of the elements of w sum to c (a perfect packing of the knapsack).

(d) The set of pairs (i, j) of nonnegative integers written in binary such that $\varphi(i, j)$ holds but $\varphi(i', j)$ does not hold for $0 \leq i' < i$, where the predicate φ is computable in time polynomial in the length of its inputs written in binary.

(e) The set of valid computations of a universal Turing machine.

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