Computer Science Department
Stanford University
Comprehensive Examination in Numerical Analysis
Autumn 1995

October 20, 1995

READ THIS FIRST!

1.   You should write your answers for this part of the Comprehensive Examination in a
BLUE BOOK. Be sure to write your MAGIC NUMBER on the cover of every blue book
that you use.

2.   This exam is CLOSED BOOK. You may not use notes, articles, or books.

3.   Show your work, since PARTIAL CREDIT will be given for incomplete answers. For
example, you can get credit for making a reasonable start on a problem even if the
idea doesn't work out; you can also get credit for realizing that certain approaches
are incorrect. On a true/false question, you might get partial credit for explaining
why you think something is true when it is actually false. But no partial credit can
be given if you write nothing.

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(Problem I)

(15 points). Linear Algebra

(a) Let A be a real $n \times n$ symmetric matrix with n distinct real eigenvalues. Show that the eigenvectors of A are orthogonal to one another. If A is also positive definite show that the eigenvalues are positive.

(b) Define the Euclidean norm of a vector in R^n.

(c) Let A be a real, positive-definite, $n \times n$ symmetric matrix with eigenvalues $0 < \lambda_1 < \ldots < \lambda_n$. Let

$$\|x\|_A^2 = x^T A x.$$

Prove that $\| \cdot \|_A$ as defined is indeed a norm on R^n.

(Problem II)

(15 points). Differential Equations and Quadrature

(a) Define the composite trapezoidal rule for the approximate integration of

$$I := \int_a^b f(x)dx$$

over n intervals of equal length $h = (b - a)/n$. State the order of accuracy of the method in terms of h, under an assumption on the smoothness of f which you should state.

(b) Use the quadrature rule derived in (a) to derive a numerical approximation to the differential equation

$$\frac{du}{dt} = f(t), \quad u(\tau) = u_0,$$

at time $t = \tau + 1$, by partitioning the interval $[\tau, \tau + 1]$ into n equal subintervals.

(c) How big (in terms of h) would you expect the error to be if you applied the method in (b) to the equation

$$\frac{du}{dt} = t^{3/2}, \quad u(\tau) = u_0,$$

with $\tau \geq 0$. What are the practical implications of using the method on this problem if τ is very small?

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