SOLUTIONS: LOGIC

Computer Science Department
Stanford University
Comprehensive Examination in LOGIC

Autumn 1995

17 October 1995

READ THIS FIRST!

1. You should mark your answers only in the ANSWER SHEET that is provided with this part of the Comprehensive Examination. Be sure to write your MAGIC NUMBER on the answer sheet.
2. This exam is OPEN BOOK.
3. There are 10 pages of the exam including the answer sheet.
4. The exam takes 30 minutes and there are 19 questions in the exam. You will receive 1 point for each CORRECT answer, 0 points for not answering, and -1/3 points for each WRONG answer. Mark "X" in at most one box for each question.

   Example:

	(A)	(B)	(C)	(D)
1.			X	

   If you decide to change your answer, erase the old marks completely or cross out with two lines, "~~X~~".

   Example:

	(A)	(B)	(C)	(D)
1.	X		~~X~~	

47

Problem 1:

The sentence "Albert will not pass this exam unless Mary passes and Jane does not pass this exam" is best represented by which of the following (where Px represents "x will pass this exam", a represents "Albert", m represents "Mary", and j represents "Jane").

(A) $\neg(Pm \wedge \neg Pj) \rightarrow \neg Pa$

(B) $(Pm \wedge \neg Pj) \rightarrow \neg Pa$

(C) $(Pm \wedge \neg Pj) \rightarrow Pa$

(D) $\neg(Pm \wedge \neg Pj) \rightarrow Pa$

Solution 1: (A)

Problem 2:

Consider the sentences

1. $(P \rightarrow P) \rightarrow P$
2. $P \rightarrow (P \rightarrow \neg P)$

Which of the following statements correctly describes the two sentences?

(A) 1 is satisfiable, but not valid; and 2 is unsatisfiable

(B) both 1 and 2 are satisfiable, but not valid

(C) 1 is valid; and 2 is unsatisfiable

(D) 1 is valid; and 2 is satisfiable, but not valid

Solution 2: (B) The model where P is true makes the first sentence true and the second sentence false; whereas the model where P is false makes the first sentence false and the second sentence true.

Problem 3:

x, y, and z are free variables. a is a constant.

	Assertion	Goal
A1	$\boxed{p(x,x)}$ *or* $q(y)$	
G2		$r(y,z)$ *and* $\boxed{p(a,z)}$

What is the result of a AG-resolution of A1 and G2?

(A)

G2		$(not\ q(y))$ *and* $r(y,z)$

(B)

G2		$(not\ q(y))$ *or* $r(y',a)$

48

(C)	G2		$q(y)$ *and* $r(y', z)$

(D)	G2		$(\textit{not } q(y))$ *and* $r(y', a)$

Solution 3: (D)

Problem 4:

Consider the following statements:

1. For any integer n, it is possible to write a theory $\mathcal{T}$ with equality such that all models of $\mathcal{T}$ have exactly n domain elements.
2. There is a first-order predicate-logic sentence $\mathcal{F}$ such that $\mathcal{F}$ is true under some interpretation with infinite domain, but false under any interpretation with finite domain.
3. It is not possible to write a theory $\mathcal{T}$ with equality such that all models of $\mathcal{T}$ have less than n domain elements, unless we have the necessary axioms for $<$.

Which statements are true?

(A) all 1, 2 and 3

(B) only 1 and 2

(C) only 1 and 3

(D) only 2 and 3

Solution 4: (B)

Problem 5:

Which of the following are unifiable?

1. $h(f(x), x)$
2. $h(y, z)$
3. $h(z, g(y))$

(A) (1,2) only

(B) (1,2) and (1,3) only

(C) (1,2) and (2,3) only

(D) (2,3) and (1,3) only

Solution 5: (B) (1,2) are unifiable using the substitution $\{y \leftarrow f(x), z \leftarrow x\}$. (1,3) are unifiable using the substitution $\{z \leftarrow f(g(y)), x \leftarrow g(y)\}$

49

Problem 6:

Consider the following statements:

1. The deductive tableau framework can be used to show non-validity of any propositional sentences.
2. The deductive tableau framework can be used to show non-validity of any predicate-logic sentences.
3. We can use the deductive tableau to show that a predicate-logic sentence $\mathcal{F}$ is not true under any interpretation.

Which statements are true?

(A) all 1, 2 and 3

(B) only 1 and 2

(C) only 1 and 3

(D) only 2 and 3

Solution 6: (C) 1 is true because we have a decision procedure for the validity of propositional sentence. 2 is false because if we could show non-validity for predicate logic, we would have a decision procedure for the validity of predicate logic. 3 is true because it is equivalent to proving that *not* F is valid.

Problem 7:

Consider the following sentence:

$$(\exists z)\left[\ (\exists y)(\forall w)\ p(y,w,z) \rightarrow (\forall w)(\exists u)\ p(w,u,z)\ \right]$$

Which of the following skolemizations preserves **validity**?

(A) $p(y, f(z,y), z) \rightarrow p(g(z), u, z)$

(B) $p(f(z), g(z,y), z) \rightarrow p(w, u, z)$

(C) $p(f(z), w, z) \rightarrow p(g(z), u, z)$

(D) $p(y, w, z) \rightarrow p(w, u, z)$

Solution 7: (C)

Problem 8:

Assume $(\mathcal{F} \vee \mathcal{G})$ is valid. Which of the following are true statements?

1. Either $\mathcal{F}$ is valid or $\mathcal{G}$ is valid, but not necessarily both
2. $\mathcal{F}$ is valid and $\mathcal{G}$ is valid
3. $\mathcal{F}$ is satisfiable or $\mathcal{G}$ is satisfiable

(A) only 1

(B) 1 and 3 only

(C) 2 and 3 only

(D) only 3

Solution 8: (D)

50

Problem 9:

Let $\mathcal{F}$ and $\mathcal{G}$ be arbitrary sentences, and define S_1 and S_2 as follows:

$$S_1 : \quad (\exists x)\mathcal{F} \rightarrow (\exists x)\mathcal{G}$$

$$S_2 : \quad (\exists x)(\mathcal{F} \rightarrow \mathcal{G})$$

Which of the following are true?

1. S_1 is equivalent to S_2
2. Assuming that S_1 and S_2 are closed, then if S_1 is valid then S_2 is valid
3. Assuming that S_1 and S_2 are closed, then if S_2 is valid then S_1 is valid

(A) only 1

(B) only 2

(C) only 3

(D) 1 and 3 only

Solution 9: (B)

Problem 10:

Consider the *nand* connective with the following semantics:
The truth-value of $\mathcal{F}$ *nand* $\mathcal{G}$ is false if both $\mathcal{F}$ and $\mathcal{G}$ are true, and true otherwise.
Which of the following NAND-splitting deduction rules preserves equivalence of a tableau?

1.

assertions	goals
A_1 *nand* A_2	
	A_1
	A_2

2.

assertions	goals
A_1 *nand* A_2	
A_1	
A_2	

3.

assertions	goals
	A_1 *nand* A_2
A_1	
A_2	

(A) only 1

(B) only 2

(C) only 3

(D) none of the above

Solution 10: (C)

51

Problem 11:

Consider a tableau deduction rule of the form:

assertions	goals
$\mathcal{A}_1$	
	$\mathcal{G}_1$
$\mathcal{A}_2$	
	$\mathcal{G}_2$

where $\mathcal{A}_1$ and $\mathcal{G}_1$ are the *required rows* and $\mathcal{A}_2$ and $\mathcal{G}_2$ are the *generated rows*.

Under which of the following assumptions is this rule sound (i.e. does it preserve the validity of the tableau)?

(A) if $\mathcal{A}_1$ is valid then $\mathcal{A}_2$ is valid, and
if $\mathcal{G}_2$ is valid then $\mathcal{G}_1$ is valid.

(B) if $\mathcal{A}_2$ is valid then $\mathcal{A}_1$ is valid, and
if $\mathcal{G}_1$ is valid then $\mathcal{G}_2$ is valid.

(C) if $\mathcal{A}_1$ is valid then $\mathcal{A}_2$ is valid, and
if $\mathcal{G}_1$ is valid then $\mathcal{G}_2$ is valid.

(D) none of the above

Solution 11: (D)

Problem 12:

Suppose Γ is a set of sentences with arbitrarily large finite models (that is, for each positive integer n, there is a model of Γ whose domain has at least n members). Which of the following are true?

1. Γ has a model whose domain is infinite
2. No infinite model satisifies Γ
3. All finite models satisfy Γ

(A) only 1

(B) 2 and 3 only

(C) only 2

(D) 1 and 3 only

Solution 12: (A) 1 is a theorem of first-order logic.

Problem 13:

Let A be a sentence of first-order logic. Which of the following are true?

1. There is an algorithm that will halt and return "yes" iff A is valid
2. There is an algorithm that will halt and return "yes" iff $\neg A$ is valid
3. There is an algorithm that will always halt, returning "yes" iff A is valid, and returning "no" otherwise

52

(A) only 1

(B) only 3

(C) only 1 and 2

(D) 1, 2, 3

Solution 13: (C) Note that 1 and 2 do not imply 3

Problem 14:

Suppose A_1 and A_2 are two (possibly infinite) sets of consistent sentences of first-order logic such that $Cn(A_1) = Cn(A_2)$ (where $Cn(\Gamma)$ =the consequences of Γ). Which of the following are true?

1. $Cn(A_1 \cup A_2) = Cn(A_1)$
2. $Cn(A_1 \cap A_2) = Cn(A_1)$
3. $\exists\varphi$ such that $Cn((A_1 \cap A_2) \cup \{\varphi\}) = Cn(A_1)$

(A) only 1

(B) only 2

(C) only 1 and 3

(D) 1, 2, 3

Solution 14: (A)

Problem 15:

Let $\mathcal{L}$ be a first-order language with equality which consists of **0**, **S** and **mod**, which are zero-, one- and two-place function symbols, and a two-place predicate symbol $<$.

Let $\Re_{mod} = (\mathcal{N}, 0, S, mod, <)$ be the intended structure of the language $\mathcal{L}$ such that $\forall$ is interpreted over the natural numbers $\mathcal{N}$, the symbol **0** is the number 0, **S** is the successor function $S(n) = n + 1$, **mod** is the modulo function *mod*, and $<$ is the usual ordering relation on $\mathcal{N}$.

Consider the following statements:

1. The set of all valid sentences of $\mathcal{L}$ is effectively enumerable.
2. The theory of $\Re_{mod}$ is decidable.

Which of the two statements are true?

(A) only 1

(B) only 2

(C) both 1 and 2

(D) neither

Solution 15: (A) 1 is true because of the Enumerability Theorem. 2 is false because we can define multiplication using *mod* and $<$.

53

Problem 16:

Consider the following statements (where $Cn(\Gamma)$ = the consequences of Γ):

1. If Σ is finitely axiomatizable, then Σ has a finite model.
2. If Σ is an infinite set and there is no finite subset $\Sigma' \subset \Sigma$ such that $\Sigma \subsetneq Cn(\Sigma')$, then $Cn(\Sigma)$ is not finitely axiomatizable.
3. If Σ is a set of sentences in a countable language and Σ is unsatisfiable by any countable structure, then Σ is unsatisfiable by structures of all cardinalities.

Which of the above statements are true?

(A) only 2 and 3

(B) only 1 and 3

(C) only 1

(D) all 1,2 and 3

Solution 16: (A) 1 is obviously false. 2 is clear from Theorem 26E in Enderton page 146. 3 is from the LST Theorem.

Problem 17:

For any sentence $\mathcal{F}$, if $\mathcal{F}'$ is the result of skolemizing $\mathcal{F}$, then which of the following statements are true?

1. $\mathcal{F} \equiv \mathcal{F}'$ is always valid.
2. $\mathcal{F} \equiv \mathcal{F}'$ is always invalid.
3. $\mathcal{F}$ is valid iff $\mathcal{F}'$ is valid.
4. If $\mathcal{F}$ is not valid, then $\{F, \neg F'\}$ is satisfiable.

(A) only 1 and 3

(B) only 2 and 3

(C) only 1 and 4

(D) only 3

Solution 17: (D)

Problem 18:

Which is a correct proof of validity by the method of falsification?

(A) *(not (if P then Q)) and (R or Q)*
T F T F T T T F

(B) *if (if P then Q) then ((P ≡ Q) or R)*
F T T T T F_T T F F

(C) *(not (P ≡ Q)) or (if P then not Q)*
F T T T F_T T T T F

54

(D)	*if*	*(if*	*not*	*P*	*then*	*R)*	*then*	*((P*	*and*	*not*	*R)*	*or*	*R)*
	F	*T*	*F*	*T*		*F*		*T*	F_T	*T*	*F*	*F*	*F*

Solution 18: (D). (A) shows satisfiability, not validity. (B) makes arbitrary choices for the truth value of P and Q, making the proof non-exhaustive. In (C), *if* should be annotated with F because we can deduce that P is T only after we know that *if* is F. (D) is the only correct proof by falsification.

55