Computer Science Department
Stanford University
Comprehensive Examination in Automata and Formal Languages
Autumn 1994

October 17, 1994

READ THIS FIRST!

1. You should write your answers for this part of the Comprehensive Examination in BLUE BOOKS. There are three problems in the exam. Be sure to write your MAGIC NUMBER on the cover of every blue book that you use.

2. The number of POINTS for each problem indicates how elaborate an answer is expected. The exam takes 30 minutes.

3. This exam is OPEN BOOK. You may use notes, articles, or books—but no help from other sentient agents such as other humans or robots.

4. Show your work, since PARTIAL CREDIT will be given for incomplete answers.

Comprehensive Exam:

Autumn 1994

Automata and Formal Languages (30 points)

Instructions: You are expected to *sketch* the main ideas in your solutions, but be very brief and avoid unnecessary detail. You are permitted to invoke any result proved in the Hopcroft-Ullman book provided you include the appropriate citation.

1. (8 points) Consider the following context-free grammar G.

$$\begin{aligned} S &\rightarrow bABaa \mid Sa \mid a \\ A &\rightarrow aB \\ B &\rightarrow baB \mid \epsilon \end{aligned}$$

Let L_A and L_B be the languages consisting of the terminal strings that can be derived from the variables A and B, respectively.

(a) [4 points] Show that L_A and L_B are regular by providing regular expressions for these two languages.

(b) [4 points] Show that $L(G)$ is regular by providing a regular expression for it.

2. (10 points) A *monotone* 2-SAT formula is a 2-CNF boolean formula $F(x_1, \ldots, x_n)$ which does not contain negated variables. For example:

$$F(x_1, x_2, x_3, x_4) = (x_1 \vee x_3) \wedge (x_2 \vee x_4) \wedge (x_1 \vee x_4) \wedge (x_2 \vee x_3).$$

It is clear that there always exists a truth assignment for the variables $x_1, \ldots, x_n$ satisfying the formula F – simply set each variable to TRUE.

Consider the following problem called MONOTONE 2-SAT: given a monotone 2-SAT formula F and a positive integer k, determine whether there exists a truth assignment satisfying F such that the number of variables set to TRUE is *at most* k.

Show that the MONOTONE 2-SAT problem is NP-hard. (**Hint:** Think about the vertex cover problem.)

3. (12 points) Consider the following decision problem:

> Given a deterministic finite state automaton (DFA) M over the alphabet $\Sigma = \{0, 1\}$, does $L(M)$ contain at least 2 strings?

Is this problem decidable? Justify your answer. (**Hint:** Think about the decision problems of deciding emptiness and finiteness of regular languages.)