Comprehensive Examination — Autumn 1992

Automata, Formal Languages and Logic (60 points)

Please solve only TWO problems out of Problems 1, 2 and 3. If you solve all three, we will consider only the best two scores. You must also solve Problems 4 and 5.

Problem 1 (10 points)

Show that

$$\left\{ \begin{array}{l} \forall x.\ [\forall y.\ q(x,y) \to p(x)], \\ \neg p(a) \end{array} \right\} \vdash \neg\forall y.\ q(a,y)$$

where p and q are unary and binary predicate symbols, respectively. You may use *modus ponens* and axioms from the following four groups:

1. Tautologies;
2. $\forall x.\ \alpha \to \alpha^x_t$, where t is substitutable for x in α;
3. $\forall x.\ (\alpha \to \beta) \to (\forall x.\ \alpha \to \forall x.\ \beta)$;
4. $\alpha \to \forall x.\ \alpha$, where x does not occur free in α.

To receive full credit, indicate for each line of the deduction either the axiom group to which it belongs or the two earlier lines to which you are applying ***modus ponens***.

Problem 2 (10 points)

Skolemize the following sentence, and state whether the resulting formula preserves satisfiability or validity.

$$\overset{\forall}{\forall} x.\overset{\exists}{\exists} y.(\overset{\forall}{\exists} z.\ p(x,y,z) \to q(a,x,y))$$

Problem 3 (10 points)

Let $\prec_1$ and $\prec_2$ be binary relations over some domain, and define

$$x \prec_\vee y \quad \equiv \quad x \prec_1 y \vee x \prec_2 y.$$

Determine which (if any) of the following statements is true. Justify your answer.

a. If $\prec_1$ and $\prec_2$ are well-founded, then $\prec_\vee$ is well-founded.
b. If $\prec_\vee$ is well-founded, then $\prec_1$ and $\prec_2$ are well-founded.

Problem 4 (20 points)

Are the following problems decidable or not? Prove your answer.

a. Given a Turing machine M (with a semi-infinite tape) and a specific input string w, does M ever scan a tape cell more than once when started on input w?
b. Given a Turing machine M (with a semi-infinite tape) and a specific input string w, does M ever scan the LEFTMOST tape cell more than once when started on input w?

Problem 5 (20 points)

Reduce (directed or undirected, as you prefer) HAMILTONIAN CYCLE to SAT. A clear outline of the (polynomial time) reduction will suffice.

6