Computer Science Department
Stanford University
Comprehensive Examination in Analysis of Algorithms
Autumn 1992

October 19, 1992

You should write your answers for this part of the Comprehensive Examination in BLUE BOOKs. There are 3 problems in the exam.

Be sure to write your MAGIC NUMBER on the cover of every blue book that you use.

Show your work, since PARTIAL CREDIT will be given for incomplete answers.

COMPS and SOLUTIONS 1992

AA PROBLEMS

Note: The exam is open book. One hour

20 points each question

Problem 1. Given two points on the plain, $a = (x_1, y_1)$ and $b = (x_2, y_2)$, the *Manhattan distance* between a and b is given by $d(a, b) = |x_1 - x_2| + |y_1 - y_2|$.

Suppose you are given n points, $p_1, \ldots, p_n$, on the plane. and you want to find a point q such that

$$\sum_{1 \leq i \leq n} d(p_i, q)$$

is minimized. (The desired point does not have to be one of the input points.)

Give an algorithm to solve this problem. Explain why your algorithm is correct. Your algorithm should be as fast as possible.

Problem 2. Let $G = (V, E)$ be an undirected graph. Given $V' \subseteq V$, a *subgraph induced by* V' is (V', E') where $E' = \{\{v, w\} \in E | v, w \in V'\}$. *A coloring* of G is a labeling of V by integers (colors) such that for every edge. its endpoints have different colors.

Suppose a graph G has the property that every subgraph induced by a subset of nodes has the average degree below 7. (The average degree is the sum of node degrees divided by the number of nodes.) Give a polynomial-time algorithm to color G in 7 colors. (For this problem, you do not have to give the fastest algorithm to get full credit.)

Problem 3. How many n-digit binary sequences that have no adjacent 0's are there?